BRONX COMMUNITY COLLEGE  
of the City University of New York  

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE  

MATH 30  
Nikos Apostolakis  

Midterm  
March 25, 2010  

Name: **KEY**  

**Directions:** Write your answers in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. This exam has a total of 100 points.  

1. Find the domain for each of the following functions:  

   (a) (10 points) \( f(x) = \frac{2x - 1}{x^2 - x - 12} \)  
   
   We need \( x^2 - x - 12 \neq 0 \)  
   
   Now \( x^2 - x - 12 = (x - 4)(x + 3) \).  
   
   So we need \( x \neq 4 \) and \( x \neq -3 \).  
   
   \( \therefore \) the domain is \( (-\infty, -3) \cup (-3, 4) \cup (4, \infty) \).  

   (b) (10 points) \( g(x) = \sqrt{x^2 - 4} \)  
   
   We need \( x^2 - 4 \geq 0 \). Now  
   
   \( x^2 - 4 = (x - 2)(x + 2) \). So the domain is \( (-\infty, -2) \cup [2, \infty) \).  

   (c) (10 points) \( h(x) = \log_2(9 - x^2) \)  
   
   We need \( 9 - x^2 > 0 \). Now  
   
   \( 9 - x^2 = (3 + x)(3 - x) \). So we have the following table:  

   \( \therefore \) the domain is \( (-3, 3) \).
2. (10 points) Find $f \circ g$, where $f(x) = \frac{2x - 3}{5x + 2}$ and $g(x) = \frac{x - 1}{x + 2}$

First the formula:

$$(f \circ g)(x) = f(g(x)) = \frac{2\left(\frac{x-1}{x+2}\right) - 3}{5\left(\frac{x-1}{x+2}\right) + 2} \cdot \frac{x+2}{x+2}$$

$$= \frac{2(x-1) - 3(x+2)}{5(x-1) + 2(x+2)}$$

$$= \frac{2x - 2 - 3x - 6}{5x - 5 + 2x + 4}$$

$$= \frac{-x - 8}{7x - 1} = \frac{-x - 8}{7x - 1}$$

Now the domain.

$x$ is in the domain of $f \circ g$, when the following two conditions hold:

1) $x$ is in domain of $g \iff x + 2 \neq 0 \Rightarrow x \neq -2$

2) $g(x)$ is in domain of $f \iff 7x - 1 \neq 0 \Rightarrow x \neq \frac{1}{7}$

Combining these two conditions we have: $\quad -\infty \quad -2 \quad \frac{1}{7} \quad +\infty$

domain is $\quad (-\infty, -2) \cup (-2, 1/7) \cup (1/7, \infty)$,
3. (10 points) Let \( f(x) = x^2 - 4x - 2 \) with domain \((-\infty, 2]\), and \( g(x) = 2 - \sqrt{x+6} \). Prove that \( f \) and \( g \) are a pair of inverse functions.

We will show that:

a) \( f \) is in the domain of \( g \), \( f(g(x)) = x \).

\[
f(g(x)) = 2 - \frac{(2-\sqrt{x+6})^2}{4} - 2 \quad \text{ Now if } x \text{ is in the domain of } f, x - 2 < 0 \text{. So } |x-2| = -(x-2).
\]

\[
= 4 - 4 \sqrt{x+6} + x+6 - 8 + 4 \sqrt{x+6} - 2
\]

\[= x.\]

b) \( g \) is in the domain of \( f \), \( g(f(x)) = x \).

\[
g(f(x)) = 2 - \frac{\sqrt{(x^2-4x-2)+6}}{x-2}
\]

\[= 2 - \frac{\sqrt{(x^2-4x+4)}}{x-2}
\]

\[= 2 - \frac{\sqrt{(x-2)^2}}{x-2}
\]

\[= 2 - 1 \quad x - 2 \quad \text{So } g(f(x)) = 2 - (x + 2)
\]

\[= x.\]

4. (10 points) Find the formula, the domain and the range of \( f^{-1} \), where

\[
f(x) = \frac{-x+3}{4x-7}
\]

So \( f^{-1} \) is

\[
x = \frac{-y+3}{4y-7}
\]

We solve for \( y \):

\[x(4y-7) = -y+3
\]

\[4xy - 7x = -y + 3
\]

\[4xy + y = 7x + 3
\]

\[(4x+1)y = 7x + 3
\]

\[y = \frac{7x+3}{4x+1}
\]

Thus the formula is

\[f^{-1}(x) = \frac{7x + 3}{4x+1}
\]

Domain of \( f^{-1} \): We need \( 4x+1 \neq 0 \).

So \( x \neq \frac{-1}{4} \). Thus domain is \((-\infty, \frac{-1}{4}) \cup (\frac{-1}{4}, \infty)\).

Range of \( f^{-1} \) is the domain of \( f \).

\[
4x - 7 \neq 0 \quad \text{So } x \neq \frac{7}{4}
\]

Range is \((-\infty, \frac{7}{4}) \cup (\frac{7}{4}, \infty)\).
5. (5 points) List all possible rational roots of the following polynomial, according to the “Rational Root Theorem”.

\[ p(x) = 6x^5 - 3x^4 + 7x^3 - 2x^2 + 8x - 12 \]

Possible numerators: \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \)
Possible denominators: \( \pm 1, \pm 2, \pm 3, \pm 6 \)
So possible solutions (after eliminating repetitions):
\( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6} \)

6. (15 points) Solve the following equation:

\[ x^5 - 5x^4 - x^3 + 11x^2 - 6 = 0 \]

We try them using synthetic division:

\[
\begin{array}{c|ccccccc}
1 & -5 & -1 & 11 & 0 & -6 \\
& & -1 & -4 & -5 & 6 & 6 & 0 \\
\hline
1 & -4 & -5 & 6 & 6 & 0 \\
& & 1 & -3 & -5 & 6 & 6 & 0 \\
\hline
1 & -3 & -8 & -2 & -2 & 0 & 0 & 0 \\
& & 1 & -5 & 0 & 0 & 0 & 0 \\
\end{array}
\]

So the L.H.S. factors as:
\[ (x-1)(x+1)^2(x^2-6x+6) \]

So we have:
\[ x = 1 \text{ or } x = -1 \quad \text{(double sol.)} \]

or
\[ x^2 - 6x + 6 = 0 \]

\[ D = 36 - 24 = 12 \]

Solutions:
\[ x = \frac{6 \pm \sqrt{12}}{2} = \frac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3} \]

In summary, \( x = 1 \) or \( x = -1 \)

or \( x = 3 + \sqrt{3} \)

or \( x = 3 - \sqrt{3} \)
7. (20 points) Solve the following inequality:

\[ x^4 - 4x^3 + 3x^2 + 4x - 4 \geq 0 \]

First we factor. We check possible rational roots: \( \pm 1, \pm 2, \) and \( \pm 4 \)

\[
\begin{array}{c|cccc}
& 1 & -4 & 3 & 4 \\
\hline
1 & 1 & -3 & 0 & 4 \\
1 & 1 & -2 & \cancel{2} & \times \\
1 & 1 & -2 & \cancel{2} & \times \\
1 & 1 & -2 & \cancel{2} & 4 \\
\end{array}
\]

So \( x^4 - 4x^3 + 3x^2 + 4x - 4 \)

\[= (x-1)(x+1)(x^2 - 4x + 4)\]
\[= (x-1)(x+1)(x-2)^2\]

So solution is \((-\infty, -1] \cup [1, \infty)\)