BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 23 | Exam 2
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Name: ANSWERS

> **Directions:** Write your answers in the provided space. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly.

1. The probability distribution of a discrete random variable X is given in the table below.

x	$P(x)$
0	0.40
1	0.15
2	0.25
3	0.20

(a) Compute the expected value (the mean) of X.

	x	$P(x)$	$x\,p(x)$	x^2	$x^2\,p(x)$
	0	0.4	0	0	0
	1	0.15	0.15	1	0.15
	2	0.25	0.5	4	1
	3	0.20	0.6	9	1.8
Σ		1	1.25		2.95

$$\mu = E(X) = \sum x\,P(x) = \boxed{1.25}$$

(b) Compute the standard deviation of X.

$$\sigma^2 = \sum x^2 P(x) - \mu^2$$
$$= 2.95 - 1.25^2$$
$$= 2.95 - 1.5625 = 1.3875$$

$$\sigma = \sqrt{1.3875} \approx \boxed{1.779}$$

Binomial with $p = 0.75$ $n = 20$ $q = 0.25$

2. 75% of the residents of Pleasantville like banana splits. If we randomly select 20 people from Pleasantville:

(a) How many of those selected we expect to like banana splits?

~~From the PDF table~~

~~$P(X=$~~

$$E(X) = n \cdot p$$

$$= 20 \cdot 0.75$$

$$= \boxed{15}$$

(b) What is the standard deviation?

$$\sigma = \sqrt{\mu \cdot q}$$

$$= \sqrt{15 \cdot 0.25}$$

$$= \sqrt{3.75}$$

$$\approx \boxed{1.936}$$

(c) What is the probability that exactly 15 of the selected people like banana splits?

From the PDF tables

$$P(X = 15) = \boxed{0.2023}$$

(d) What is the probability that more than 13 but at most 18 of the selected people like banana splits?

BACK

We have

$$P(13 < X \le 18) = P(X \le 18) - P(X \le 13)$$

From CDF $= 0.9757 - 0.2142$

$= \boxed{0.7615}$

3. Let X be a random variable that represents the length of time it takes a student to complete an exam. It was found that x has an approximately normal distribution with mean $\mu = 2.5$ hours and standard deviation $\sigma = 0.8$ hours.

(a) What is the probability that a randomly selected student takes at least 4.1 hours to complete the exam?

We want $P(X \geq 4.1)$

Using $z = \frac{x - \mu}{\sigma}$

we have

$$X = 4.1 \Rightarrow z = \frac{4.1 - 2.5}{0.8} = \frac{1.6}{0.8} = 2$$

So $P(X \geq 4.1) = P(Z \geq 2.0)$

$= P(Z \leq -2)$

$= \boxed{0.02275}$

(b) Suppose 25 students are selected at random. What is the probability that $\bar{x}$, the mean time of completing the exam for these 25 students, is not more than 2.3 hours?

Since X is n.d

$\bar{X}$ is n.d. with $\mu_{\bar{X}} = 2.5$ and $\sigma_{\bar{X}} = \frac{0.8}{\sqrt{25}} = 0.16$

BACK

$$\bar{X} = 2.3 \Rightarrow Z = \frac{2.3 - 2.5}{0.16}$$

$$= \frac{-0.2}{0.16}$$

$$= -1.25$$

So $P(\bar{X} \leq 2.3) = P(Z \leq -1.25)$

$$= \boxed{0.10565}$$

4. Colette is self-employed, selling cosmetics at home parties. She wants to estimate the average amount a client spends per year at these parties. A random sample of 16 receipts had a mean of $\bar{x} = \$340.70$ with a standard deviation of $s = \$60.15$. Find a 90% confidence interval for the mean amount μ spent by all clients. Assume x has an approximately normal distribution.

We have a small sample $(n=16)$ drawn from a n.d. population. So we use t-distribution with $r = 16-1 = 15$ degrees of freedom, with $c = 0.90$

From the tables we have $t_{0.90} = 1.753$

The error is then

$$E = t_{0.90} \frac{60.15}{\sqrt{16}}$$

$$= 1.753 \frac{60.15}{4}$$

$$= 1.753 \cdot 15.0375$$

$$\approx 26.36$$

So the 90% confidence interval is

$$340.70 - 26.36 \leq \mu \leq 340.70 + 26.36$$

that is

$$\boxed{314.34 \leq \mu \leq 367.06}$$

5. Jorge lives in Pleasantville and hates banana splits. He can't believe that 75% of his fellow residents like that stuff. He decides to test the hypothesis $H_0\colon\ p = 0.75$ with alternative hypothesis $H_a\colon\ p < 0.75$. In a random sample of 100 residents he finds that 73 like banana splits.

Is this sufficient evidence to reject H_0 at the level of significance $\alpha = 0.05$?

We have $\hat{p} = \frac{73}{100} = 0.73$, and $\hat{q} = \frac{27}{100} = 0.27$.

Since $n \cdot \hat{p} = 73$ and $n \cdot \hat{q} = 27$ both larger than 5 this sample is sufficiently large.

We have $\sigma_{\hat{p}} = \sqrt{\frac{p \cdot q}{n}} = \sqrt{\frac{0.75 \cdot 0.25}{100}} \approx 0.0433$ and so the test statistic is

$$z = \frac{0.73 - 0.75}{0.0433}$$

$$\approx -0.46$$

Two alternative ways of proceeding:

P-value

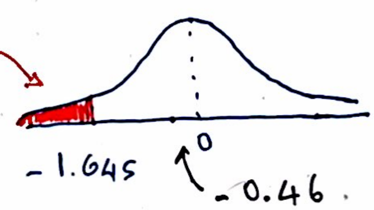
$P(z \leq -0.46) = 0.32997$

Since p-value $> \alpha = 0.05$ there is not enough evidence to reject H_0

Rejection region

This is a one-tailed test (left-tailed). From the table of t-values for $\nu = \infty$ we have $z_{0.05} = 1.645$

So we have the rejection region



Test statistic is outside rejection region.

$\boxed{\text{There is not enough evidence to reject } H_0.}$