BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MTH 23.5 Exam 2
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Name: ______________________________

Directions: Write your answers in the provided space. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly.

1. The probability distribution of a discrete random variable X is given in the table below.

x	$P(x)$
0	0.30
2	0.20
3	0.40
4	0.10

(a) Compute the expected value (the mean) of X.

(b) Compute the standard deviation of X.

2. 20% of the residents of Pleasantville like pineapple in their pizza. If we randomly select 30 people from Pleasantville:

   (a) How many of those selected we expect to like pineapple in their pizza?

   (b) What is the standard deviation of the number of people in the sample that like like pineapple in their pizza?

   (c) What is the probability that exactly 13 of the selected people like pineapple in their pizza?

   (d) What is the probability that more than 8 but at most 15 of the selected people like pineapple in their pizza?

3. Scores in a standardized test are normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 100$.

   (a) What is the probability that a randomly selected student scored at least 660 in that test?

   (b) Suppose 25 students are selected at random. What is the probability that $\bar{x}$, the mean of their scores in that test, is between 400 and 650?

4. Janele wants to know the average (mean) time that takes her to commute from her home to school. She records the time it takes her for a year, and then randomly selects 16 days. She found that the mean time it took her to commute from home to school for those selected days was $\bar{x} = 45$ minutes with a standard deviation $s = 4.5$ minutes. Construct a 95% confidence interval for the mean time that it takes Janele to commute from her home to school. Assume that the commuting time X is normally distributed.

Formulas

Discrete Random Variables

The mean or expected value:

$$\mu = E(X) = \sum x\,P(x).$$

The variance

$$\begin{aligned}\sigma^2 &= \sum (x-\mu)^2 \\ &= \sum x^2 P(x) - \mu^2\end{aligned}$$

The standard deviation

$$\sigma = \sqrt{\sigma^2}.$$

Binomial Distribution

If X is binomial distribution with n tries, probability of success p and probability of failure $q = 1 - p$, then

$$\mu = E(X) = np,$$

and

$$\begin{aligned}\sigma &= \sqrt{npq} \\ &= \sqrt{\mu q}.\end{aligned}$$

Sampling distribution

If X has mean μ and standard deviation σ, then the sample mean $\bar{X}$ for samples of size n has

$$\mu_{\bar{X}} = \mu, \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}.$$

If X is normally distributed or $n \geq 30$ then $\bar{X}$ is normally distributed.

For samples of size n, drawn from a population with proportion p, the sample proportion $\hat{P}$ has

$$\mu_{\hat{P}} = p, \quad \sigma_{\hat{P}} = \sqrt{\frac{pq}{n}}.$$

If $np > 5$ and $nq > 5$ then $\hat{P}$ is normally distributed.

Confidence intervals

The c-confidence interval for the mean μ is

$$\bar{x} - E < \mu < \bar{x} + E$$

For $n \geq 30$

$$E = z_c \cdot \sigma_{\bar{X}} = z_c \cdot \frac{\sigma}{\sqrt{n}}$$

For $n < 30$

$$E = t_c \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom $\nu = n - 1$.

If $n\hat{p} > 5$ and $n\hat{q} > 5$ then the c-confidence interval for the proportion p is

$$\hat{p} - E < p < \hat{p} + E$$

where,

$$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}.$$

Hypothesis testing

If we test for μ then the test statistic is

For $n \geq 30$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{X}}} = \frac{\bar{x} - \mu}{\sqrt{\frac{\sigma}{n}}}$$

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For $n < 30$

$$t = \frac{\bar{x} - \mu}{\sqrt{\frac{s}{n}}}$$

degrees of freedom $\nu = n - 1$.

If we test for p then the test statistic is

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$