10/16/2025

Yesterday: Synthetic division: how to divide by linear binomials of the form $x-a$.

Remainder Th^m The remainder of the division by $x-a$ is $f(a)$, for f a polynomial.

Ex $f(x) = x^5 - 2x^4 + 3x^2 - 7$. Use syntetic division to evaluate $f(2)$, $f(-3)$, $f(\frac{1}{2})$.

Sol. Divide by $x-2$, $x+3$, $x - \frac{1}{2}$

$f(2) = 5$

$f(-3) = 371$

$$\begin{array}{r|rrrrrr}
-3 \;\; 2 & 1 & -2 & 0 & 3 & 0 & -7 \\
 & & -3 & 15 & -45 & -126 & 378 \\
 & & 2 & 0 & 0 & 6 & 12 \\
\hline
 & 1 & 0 & 0 & 3 & 6 & 5 \\
 & 1 & -5 & 15 & -42 & -126 & \boxed{371}
\end{array}$$

$-4/2 \qquad \frac{24}{8} \qquad -324/1$

$f(\frac{1}{2}) = -\frac{303}{32}$.

$$\begin{array}{r|rrrrrr}
1/2 & 1 & -2 & 0 & 3 & 0 & -7 \\
 & & \frac{1}{2} & -\frac{3}{4} & -\frac{3}{8} & \frac{21}{16} & \frac{21}{32} \\
\hline
 & 1 & -\frac{3}{2} & -\frac{3}{4} & \frac{21}{8} & \frac{21}{16} & \boxed{\cancel{\frac{345}{32}}}
\end{array} \quad \boxed{-\frac{303}{32}}$$

Factor polynomials (Solving polynomial equations)

leading coefficient · constant term

Def $f(x) = \boxed{a_n} x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + \boxed{a_0}$

Thm $f(a) = 0$ iff $x - a$ is a factor of $f(x)$.

Rational root Thm If $f(x)$ has integer coefficients then if a is integer and $f(a) = 0$ the a divides a_0 (the constant term).

If $a = \frac{p}{q}$, then p divides a_0

q divides a_n.

— · —

Ex Solve $x^3 + 4x^2 - 11x - 30 = 0$

Ans Candidates roots $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

✗ Try 1

✗ -1

✓ -2

-2 -11	1	4	-11	-30
		-2	-4	30
				14
		-1	-3	
		1	5	-6
	1	5	-6	-36 ✗
	1	3	-44	-16 ✗
	1	2	-15	$\boxed{0}$

$x^3 + 4x^2 - 11x - 3 = 0 \Leftrightarrow (x+2)(x^2 + 2x - 15) = 0$

$\Leftrightarrow (x+2)(x+5)(x-3) = 0$

[redacted]

$\Leftrightarrow \boxed{x = -2 \text{ or } x = -5 \text{ or } x = 3}$

Ex 2 solve $\boxed{2}x^3 - x^2 + 5x \boxed{+3} = 0$

Solution Candidates Numerators $\pm 1, \pm 3$

Denominators $\pm 1, \pm 2$

$\underbrace{\pm 1, \pm 3}$ $\pm \frac{1}{2}, \pm \frac{3}{2}$

None of these works

$$\begin{array}{r|rrrr} -3 \quad 1 & 2 & -1 & 5 & 3 \\ & & -6 & 21 & \\ & & 2 & 1 & 6 \\ \hline & 2 & 1 & 6 & 9 \times \\ & 2 & -7 & 26 & \times \end{array}$$

$$\begin{array}{r|rrrr} \times \ \frac{1}{2} & 2 & -1 & 5 & 3 \\ & & 1 & 0 & 5/2 \\ \hline & 2 & 0 & 5 & \times \end{array}$$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -1 & 5 & 3 \\ & & -1 & 1 & -3 \\ \hline & \boxed{2 \quad -2 \quad 6} & & & \boxed{0} \end{array}$$

quotient

$x = -\frac{1}{2}$ OR $2x^2 - 2x + 6 = 0$

$\frac{2x^2 - 2x + 6 = 0}{2} \iff x^2 - x + 3 = 0$

$x = \frac{-b \pm \sqrt{D}}{2a}$

$D = b^2 - 4ac$
$= 1 - 4 \cdot 1 \cdot 3$
$= \boxed{-11} < 0$

No solutions.

So only real solution is $\boxed{x = -\frac{1}{2}}$

Ex 3 Solve $\boxed{3}x^4 - 5x^3 - 41x^2 - 43x \boxed{-10} = 0$

Solution Candidates: Numerators $\boxed{\pm 1, \pm 2, \pm 5, \pm 10}$

Denominators $\pm 1, \pm 3$

$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}$

$$
\begin{array}{r|rrrrr}
-1 & 3 & -5 & -41 & -43 & -10 \\
 & & -3 & 8 & 33 & 10 \\
 & & 3 & -2 & -43 & \\
\hline
 & 3 & -2 & -43 & -46 & \times \\
 & \boxed{3} & \boxed{-8} & \boxed{-33} & \boxed{-10} & \boxed{0}
\end{array}
$$

$x = -1$ is a solution.

$$3x^3 - 8x^2 - 33x - 10 = 0$$

$$
\begin{array}{r|rrrr}
-1 & 3 & -8 & -33 & -10 \\
 & & -3 & 11 & \\
\hline
 & 3 & -11 & -22 & \times
\end{array}
$$

so -1 not double

$$
\begin{array}{r|rrrr}
2 & 3 & -8 & -33 & -10 \\
 & & 6 & -4 & \\
\hline
 & 3 & -2 & -37 & \times
\end{array}
$$

$x = -2$ ✓

$$
\begin{array}{r|rrrr}
-2 & 3 & -8 & -33 & -10 \\
 & & -6 & 28 & 10 \\
\hline
 & 3 & -14 & -5 & \boxed{0}
\end{array}
\checkmark
$$

3 coefficients $\Rightarrow$ quadratic

$$3x^2 - 14x - 5 = 0$$

$x=-1,\ x=-2$ or $3x^2-14x-5=0$

$$x = \frac{14 \pm \sqrt{316}}{6} = \frac{\overset{7}{\cancel{14}} \pm \cancel{2}\sqrt{79}}{\cancel{6}\,3} = \boxed{\frac{7 \pm \sqrt{79}}{3}}$$

$$D = 196 - \overset{120}{4 \cdot 3 \cdot (-5)}$$
$$= 316$$

5

So $\boxed{x=-1,\quad x=-2,\quad x=\frac{7+\sqrt{79}}{3},\quad x=\frac{7-\sqrt{79}}{3}}$

$$\boxed{a_n} x^n + a_{n-1}x^{n-1} + \dots + a_1 x \boxed{+\ a_0} = 0$$

- Candidate rational roots

  Numerators divisors of a_0

  Denominators divisors of a_n

- Try all candidates until we find a root (if not we give up)

- Keep going with the quotients until we get to a quadatic equation.

- Use quadratic formula