September 25

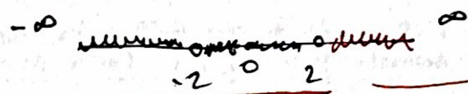
Ex. Find the domain of the following functions

a) $f(x) = \dfrac{2x}{x^2-4}$ b) $f(x) = \dfrac{x^2+5}{x^2+2x+1}$

c) $f(x) = \sqrt{3x-5}$ d) $f(x) = \sqrt{x^2+4}$

Solution a) Denominators need to be non-zero. So we need $x^2-4 \neq 0$. We need to exclude solutions of $x^2-4=0 \iff x^2=4 \iff \boxed{x=\pm 2}$

Domain $x \neq \pm 2$




$\boxed{(-\infty,-2) \cup (-2,2) \cup (2,\infty)}$

$(x+1)^2 = 0$

b) Need $x^2+2x+1 \neq 0$. Bad values of $x$: $x^2+2x+1=0$

$$\boxed{\begin{array}{l} x = \dfrac{-b \pm \sqrt{D}}{2a}, \quad D = b^2-4ac \\ \text{Solutions of } ax^2+bx+c=0 \end{array}}$$

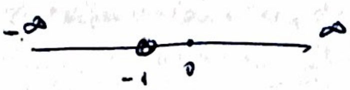
$a=1$
$b=2$
$c=1$

$x = \dfrac{-2 \pm \sqrt{0}}{2} = -1$

$D = 2^2 - 4\cdot 1\cdot 1$
$= 0$

So Domain $x \neq -1$

$\boxed{(-\infty,-1) \cup (-1,\infty)}$




b') $g(x) = \dfrac{4}{2x^2+x+1}$ Need $2x^2+x+1 \neq 0$

Bad values $2x^2+x+1=0$ No real numbers are excluded

$x = \dfrac{-1 \pm \sqrt{-7}}{4}$ not real numbers

$\boxed{D = (-\infty,\infty)}$ $(\mathbb{R})$

$D = 1^2 - 4\cdot 2\cdot 1 = -7$ negative

c) For square roots: the randicand $\geq 0$

We need $3x - 5 \geq 0$

$\iff 3x \geq 5$

$\iff \boxed{x \geq 5/3}$

$x \geq 5/3$

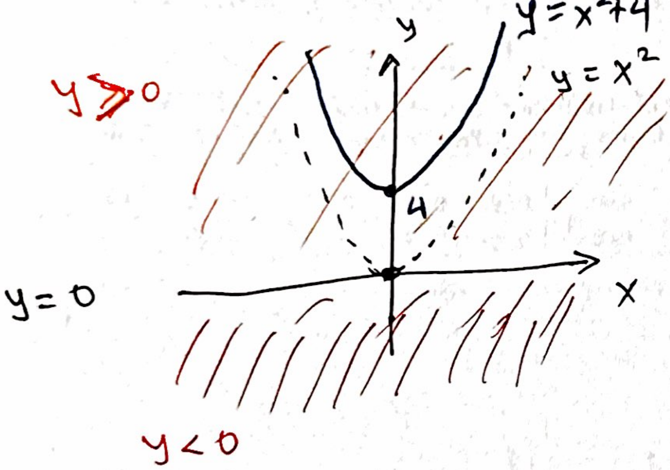
$-\infty$ $\infty$

$5/3$

$\boxed{[\frac{5}{3}, \infty)}$

d) Need $\boxed{x^2 + 4 \geq 0.}$ Always true Domain $\boxed{(-\infty, \infty)}$

Graph $y = x^2 + 4$ and see when $y \geq 0$

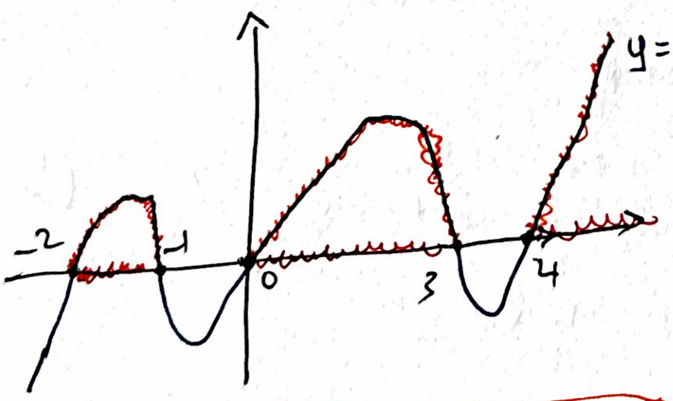



always above x-axis

y always positive

e) Find the domain of $f(x) = \sqrt{g(x)}$

where the graph of $y = g(x)$ is shown



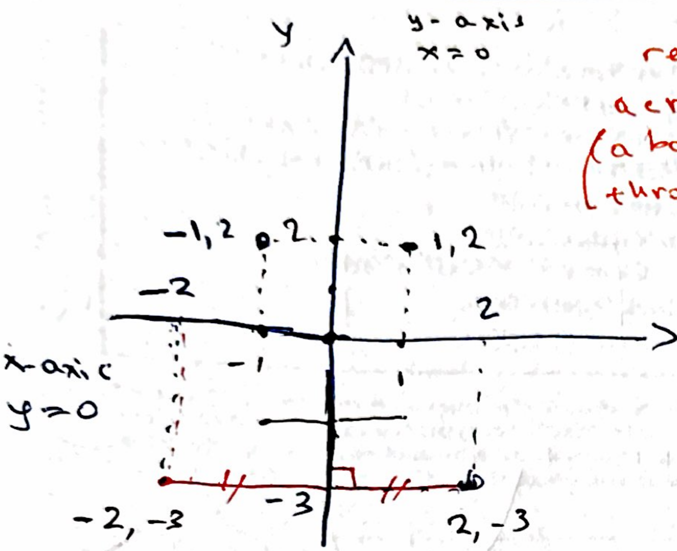

Sol. Need

$g(x) \geq 0$

$[-2, -1] \cup [0, 3] \cup [4, \infty)$

Quiz: First 20-30 min in class.

## Transformation of functions

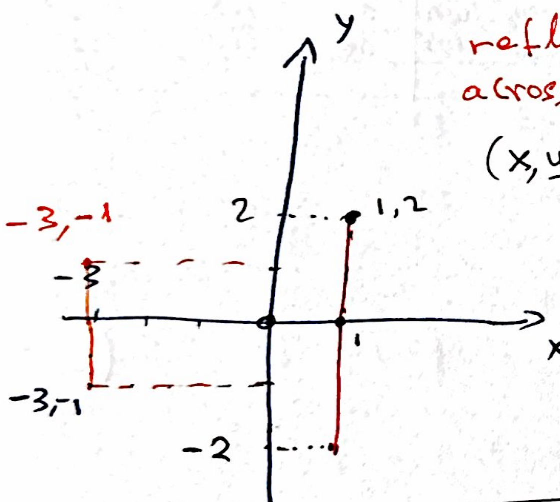



reflection across (about / through) y-axis

$x, y \longmapsto -x, y$

Ex. $(1,2) \mapsto (-1, 2)$

$(2,-3) \longmapsto (-2,-3)$

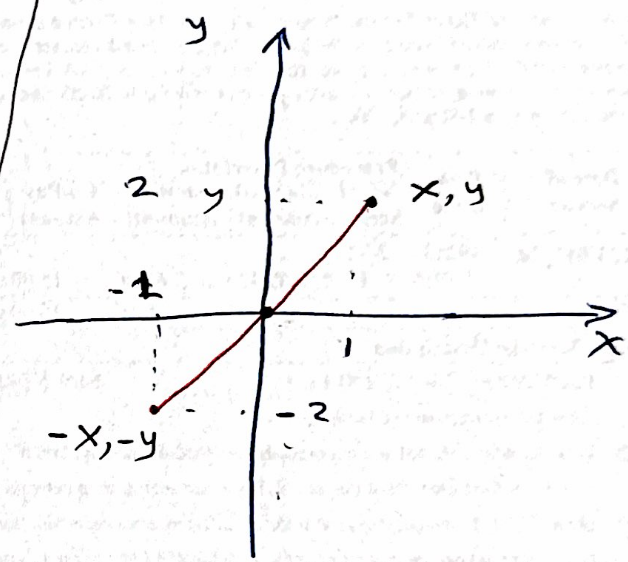



reflection across x-axis

$(x,y) \longmapsto (x,-y)$

$(-3,-1) \longmapsto$

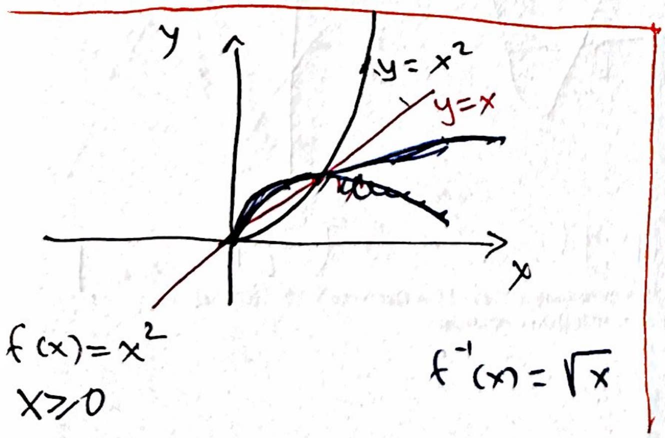
$(-3, 1)$

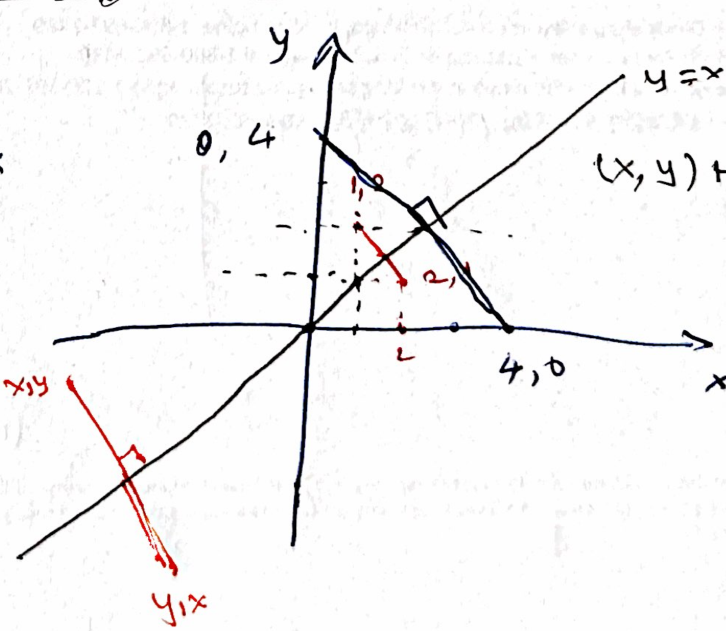



reflection across $(0,0)$

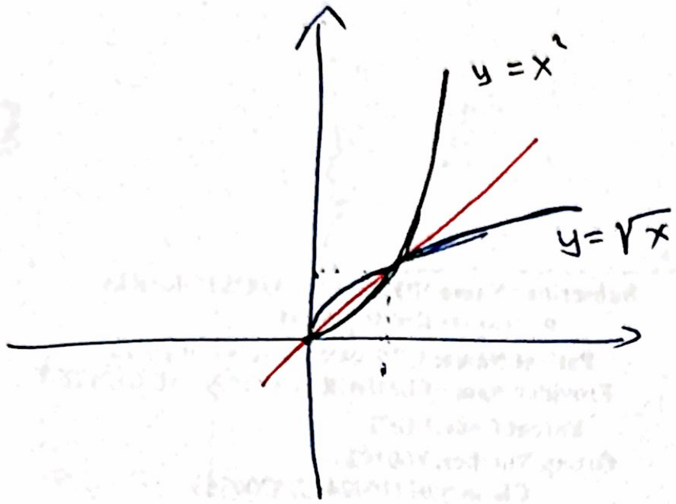
## Diagonal line $y=x$

(identity function) $f(x)=x$

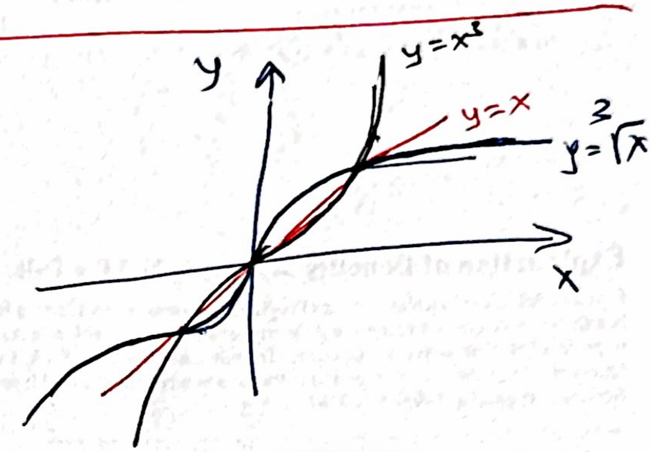



$(x,y) \longmapsto (y,x)$




The graph of $y = f^{-1}(x)$ is the reflection across the diagonal of $y = f(x)$.




$f(x) = x^3 \qquad f^{-1}(x) = \sqrt[3]{x}$

**Ex** Verify that $f(x) = \dfrac{x}{2+x}$, $g(x) = \dfrac{2x}{1-x}$ is a pair of inverse functions. State domain and range of each.

**Solution** Need to verify

$$\boxed{\begin{array}{c} f(g(x)) = x \\ \text{and} \\ \boxed{g(f(x)) = x} \end{array}}$$

You do this

$$f(g(x)) = \frac{\left(\frac{2x}{1-x}\right)}{2 + \left(\frac{2x}{1-x}\right)}$$

$$= \frac{\frac{2x}{\cancel{1-x}}}{\frac{2(1-x)+2x}{\cancel{1-x}}}$$

$$= \frac{2x}{2 \cancel{-2x} \cancel{+2x}}$$

$$= \frac{2x}{2}$$

$$= x$$

Domain of $f$ $\quad x \neq -2$

Range of $f$ $\quad y \neq 1$

Domain of $g$ $\quad x \neq 1$

Range of $g$ $\quad y \neq -2$