October 30

Exponential and Logarithmic Functions

Ex $\underline{f(x) = 2^x}$ exponential $(3^x, (1/2)^x)$ $f(x) = x^2$ polynomial

Exponential function with base 2.

Domain $\mathbb{R} = (-\infty, \infty)$

$f(1) = 2^1 = 2$

$f(2) = 2^2 = 4$

$f(3) = 2^3 = 8$

$f(0) = 2^0 = 1$

$f(-1) = 2^{-1} = 1/2$

$f(-2) = 2^{-2} = \frac{1}{2^2} = 1/4$

$f(1/2) = 2^{1/2} = \sqrt{2}$

$f(1/3) = 2^{1/3} = \sqrt[3]{2}$

$f(2/3) = 2^{2/3} = \sqrt[3]{2^2}$ exp → 2, 3 → radical

$= \sqrt[3]{4}$

$2^{-x} = \frac{1}{2^x}$

$2^{3/8} = \sqrt[8]{2^3}$ exponent → 3, radical → 8

x	$y = 2^x$	$y = (1/2)^x$
1	2	1/2
2	4	1/4
3	8	1/8
0	1	1
-1	1/2	2
-2	1/4	4
-3	1/8	8
1/2	$\sqrt{2}$	$1/\sqrt{2}$
1/3	$\sqrt[3]{2}$	⋮
2/3	$\sqrt[3]{4}$	⋮
-1/2	$1/\sqrt{2}$	

$\underline{f(x) = 2^x}$ $\underline{\text{increasing}}$

$2^{\sqrt{2}}$

$\sqrt{2} = 1.412135 6237\ldots$

$f(\sqrt{2}) = 2^{\sqrt{2}}$

$f(\pi) = 2^{\pi}$

1

$\frac{14}{10} = 1.4$

1.41

$\frac{141}{100}$ 1.412

1.4121

1.41213

$2^1 = 2$

$2^{1.4} = \sqrt[10]{2^{14}} \approx 2.639\ldots$

⋮

$2^{\sqrt{2}}$

limit of sequence

Exponential functions grow very fast.

$$b^{m+n} = b^{m} \cdot b^{n}$$

$$b^{nx} = (b^{x})^{n}$$

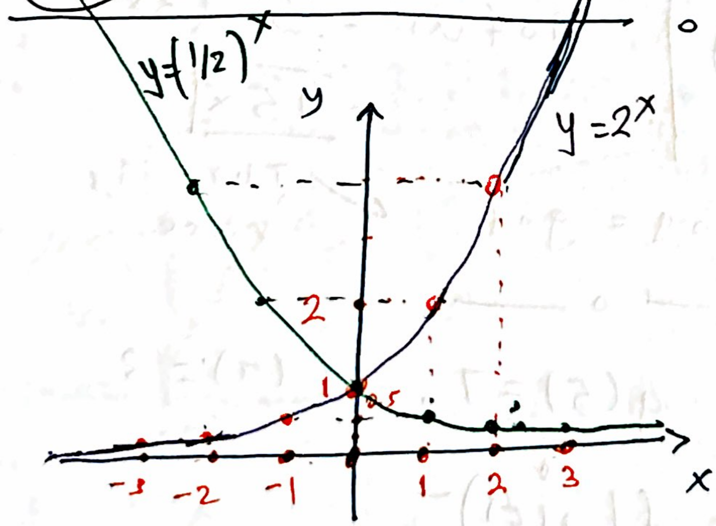
x	0	1	2	3	4	5	6	7	8	9	10	20
$y=2^x$	1	2	4	8	16	32	64	128	256	512	1024	$\sim 10^6$

$$2^{20} = (2^{10})^2 \approx (1000)^2 = 1{,}000{,}000$$

$$2^{30} = 2^{20} \cdot 2^{10} \approx 1{,}000{,}000 \times 1{,}000 = 1{,}000{,}000{,}000$$

$$2^{40} = 2^{30} \cdot 2^{10} \approx 1{,}000{,}000{,}000 \times 1000 = 1{,}000{,}000{,}000{,}000$$

$2^{63} \sim$ a sixtilion.



as $x \to -\infty$

$2^x \to 0$

as $x \to \infty$

$2^x \to \infty$

reflected across y-axis

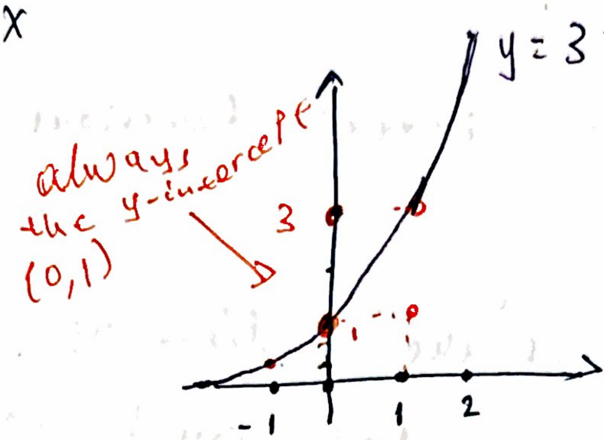
$f(x) = 2^x$ $\qquad 2^x > 0$

$f(x) = b^x$

$b = 2$

If $b > 1$ then b^x increases (exponentially)

$f(x) = 3^x$



(0, 1)

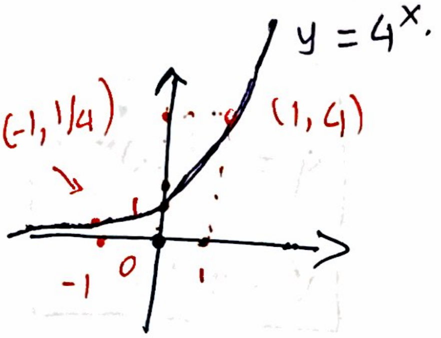
(0, b)

$f(0)=1$

$f(1)=3$

$f(2)=9$

$f(x) = 4^x$



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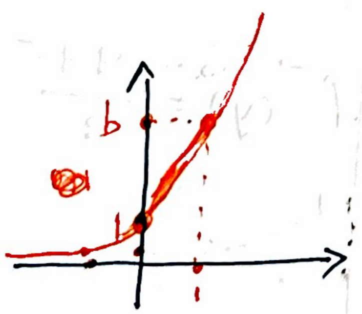
The base should be positive, if we want the domain to be $(-\infty, \infty)$. $f(x)=(-2)^x$ for this function $f(1/2)$ is undefined as a real number. bc. $(-2)^{1/2} = \sqrt{-2}$.

So $b>0$

For $b>1$ exponential growth



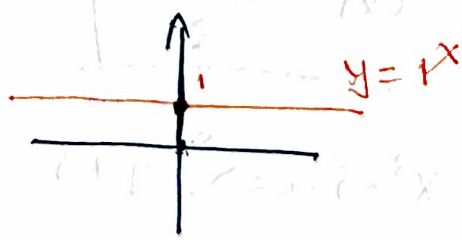
(0, 1)

(1, b)

(-1, 1/b)

For $b=1$ $f(x) = 1^x = 1$

constant

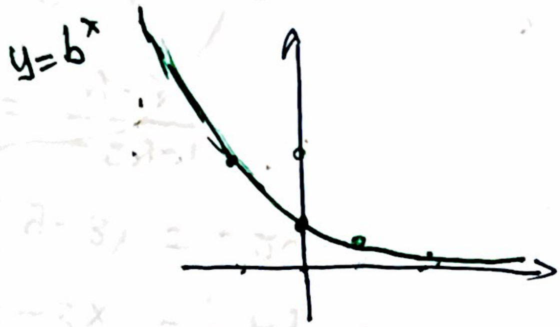


Base $b < 1$ Example $f(x) = \left(\frac{1}{2}\right)^x$

Exponential decay. $f(x)$ is decreasing really really fast.

Note $\left(\frac{1}{2}\right)^x = \frac{1}{2^x}$

$f(x) = b^x$ with $b < 1$



(0, 1)
(1, b)
(−1, 1/b)

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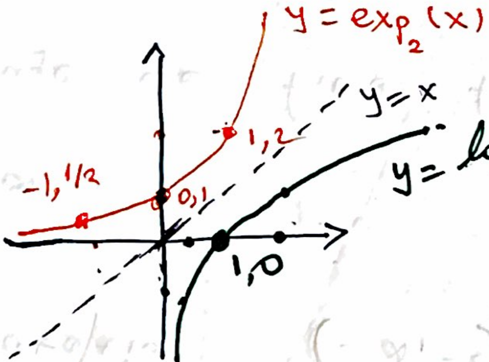
Logarithmic functions are the inverse functions of exponential functions. $b > 1$

$f(x) = 2^x$

$\exp_2(x) = 2^x$



$f(x) \nearrow \Rightarrow f(x)$ is 1-1

$x \to \infty \quad \exp_2(x) \to \infty$

$x \to -\infty \quad \exp_2 \to 0$

Def^n $\log_2$ is the inverse function of $\exp_2$

$$y = 2^x \iff x = \log_2 y$$

$\log_2 x$ is the exponent we have to raise 2 to get x. $2^3 = 8$ $\log_2 8 = 3$

exponent

$2^0 = 1$ logarithm of 1

$\log_2 1 = ?$ $2^? = 1 \Rightarrow \log_2 1 = 0$ $\log_2 4 = 2$ $2^2 = 4$