November 6

e : Euler constant (Napier)

Solve: $\ln(2x+3) = 1$ ($\log_e$)

Log equations may have "extraneous" solutions

Solution

$\ln(2x+3) = 1$

Take exp on both sides ($\exp(x) = e^x$)

$e^{\ln(2x+3)} = e^1$

$\Longleftrightarrow 2x+3 = e$

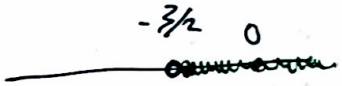
$\Longleftrightarrow 2x = e-3$

$\Longleftrightarrow x = \frac{e-3}{2}$

Domain $2x+3 > 0$

$\Longleftrightarrow 2x > -3$

$\Longleftrightarrow \boxed{x > -\frac{3}{2}}$



$(-3/2, \infty)$

Check $\frac{e-3}{2} > -\frac{3}{2}$

① calculator ✓

2nd method

$\frac{e-3}{2} \overset{?}{>} -\frac{3}{2}$

$\Longleftrightarrow e-3 > -3$ ✓ (2.718...)

Remember Domain of log is $(0, \infty)$

Ex Find the domain of

$f(x) = \log(\underbrace{-x^2+2x-1}_{>0})$

$f(x) = \ln(-x^2+2x-1)$

$\log_5(-x^2+2x-1)$

Solution Need $-x^2+2x-1>0$

To solve this we graph $y=-x^2+2x-1$

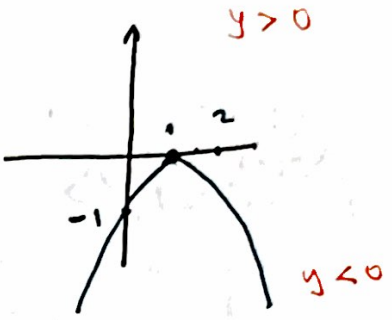
Vertex (h,k) $V=(1,0)$

$h=-\frac{b}{2a}$ $h=-\frac{2}{-2}=1$

$k=-\frac{D}{4a}$ $k=-\frac{0}{-4}=0$

$D=b^2-4ac$ $D=4-4(-1)\cdot(-1)$
$=0$

Double root $x=-\frac{b}{2a}=0$



From the graph we see that $-x^2+2x-1>0$ has no solutions. Domain $\emptyset$

This function is undefined for all real x.

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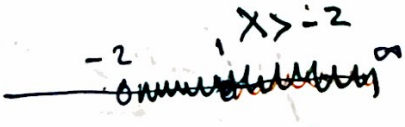
Example Solve $\ln(x-1)+\ln(x+2)=0$

solution First find domain of definition.

Need $x-1>0$ and $x+2>0$ $\Leftrightarrow$ $x>1$ and $x>-2$ $\Leftrightarrow$ $x>1$



Domain $(1,\infty)$

Condense the LHS $\quad \ln(a) + \ln(b) = \ln(ab)$

$$\ln(x-1) + \ln(x+2) = 0 \iff \ln\big((x-1)(x+2)\big) = 0$$

$$\iff \ln(x^2+x-2) = 0$$

take exp on both sides $\iff e^{\ln(x^2+x-2)} = e^0$

$$\iff x^2+x-2 = 1$$

$$\iff x^2+x-3 = 0$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{13}}{2}$$

$$\iff \boxed{x = \frac{-1+\sqrt{13}}{2}} \quad \text{or} \quad \boxed{x = \frac{-1-\sqrt{13}}{2}}$$

in domain $x > 1$

Extraneous no. in domain (e.g. calculator)

$D = 1 - 4(1)(-3) = 13$

Now check!

necessary step

$\sqrt{13} > 3$

$-1+\sqrt{13} > 2$

$\frac{-1+\sqrt{13}}{2} > \frac{2}{2} = 1$

Solution $\quad \boxed{x = \frac{-1+\sqrt{13}}{2}}$

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Example $\quad$ Solve $\log_5(3-x) = 2$

Solution $\quad$ Domain: Need $3-x > 0 \iff \boxed{3 > x}$

Take $\exp_5$: $\quad 5^{\log_5(3-x)} = 5^2 \iff \boxed{x < 3}$

$\iff 3 - x = 25$

$\iff \boxed{x = -22}$ ✓ $\quad$ check $-22 < 3$

$(-\infty, 3)$

Quiz on logs and exps this Monday 11/10

- Domain of logs.

Find the domain of

a) $f(x) = \log_2(x^2 - 7x + 12)$ b) $g(x) = 3 - \ln(-x^2+4)$

- Properties of logs and exps

Expand a) $\log(100\,x^3y^4)$ b) $\log_2 \frac{4x^2y^7}{z^8}$

Condense $3\ln x - 4\ln(y-3) + 2\ln z$

- Solve exp and log equations.

Solve a) $3^{x+1} = 4^x$

b) $\ln(3x-5) = 2$