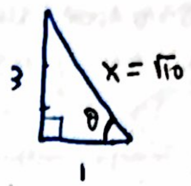
a) $\cos\left(\tan^{-1}(3)\right) = \frac{\sqrt{10}}{10}$

$3^2 + 1^2 = x^2 \Rightarrow x^2 = 10$
$\Rightarrow x = \sqrt{10}$

$\tan^{-1}(3)$ is an arc θ in $(-\pi/2, \pi/2)$ with ~~tangen~~ $\tan\theta = 3$



Since $\tan\theta > 0$, θ is actually in I : $0 < \theta < \frac{\pi}{2}$

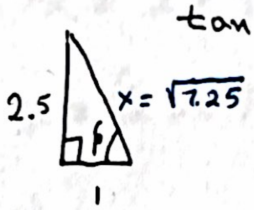
$$\boxed{\cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}}$$

—o—

b) $\sin\left(\tan^{-1}(-2.5)\right)$

$x^2 = 2.5^2 + 1^2 \Rightarrow x^2 = 7.25$
$\Rightarrow x = \sqrt{7.25}$

$\tan\varphi = 2.5$



Let $\theta = \tan^{-1}(-2.5)$.

then $\tan\theta = -2.5$

and $\underbrace{-\frac{\pi}{2} < \theta < 0}_{\text{IV}}$

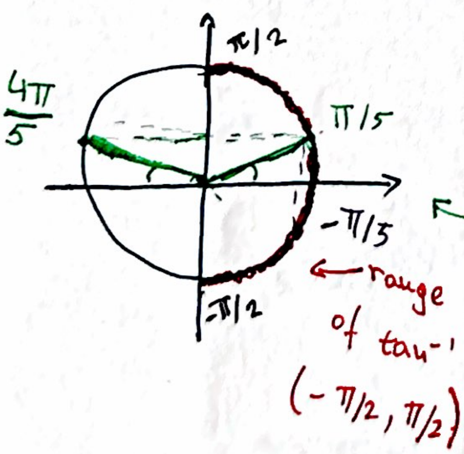
thus $\underline{\sin\theta < 0}$

$$\boxed{\sin\theta = -\frac{2.5}{\sqrt{7.25}}}$$

—o—

c) $\tan^{-1}\left(\tan\frac{4\pi}{5}\right) = -\frac{\pi}{5}$

$\frac{4\pi}{5} = \pi - \frac{\pi}{5}$ $\frac{4\pi}{5}$ is in II in II tan is negative



we are looking for an angle θ in $(-\pi/2, \pi/2)$ with $\tan\theta = \tan\frac{4\pi}{5}$

so θ $\underbrace{-\pi/2 < \theta < 0}_{\text{IV}}$

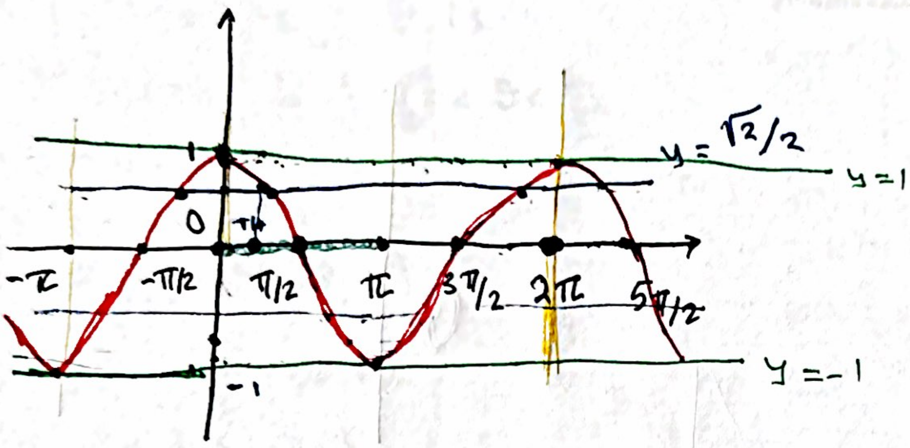
From figure $\theta = -\frac{\pi}{5}$

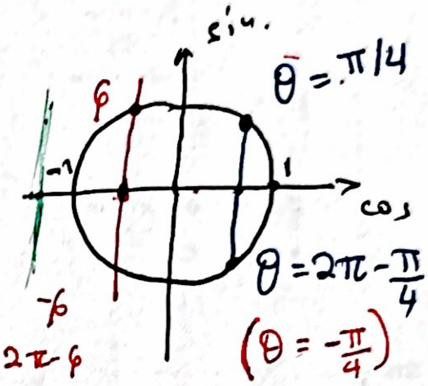
Nov 20, 2025

Trig equations

Example Solve $\cos x = \frac{\sqrt{2}}{2}$.

Ans We know one solution. $x = \frac{\pi}{4}$ $\left(\cos^{-1}\frac{\sqrt{2}}{2}\right)$





Has a unique solution in $[0, \pi]$ $x = \pi/4$

Two solutions in $[0, 2\pi)$ $x = 2\pi - \pi/4 = \boxed{\frac{7\pi}{4}}$

One solution in $[-\pi/2, 0]$ $x = -\frac{\pi}{4}$

In general in every interval of length 2π there are two solutions.

If θ is a solution $2\pi+\theta,\ 4\pi+\theta,\ 8\pi+\theta, \dots$

$-2\pi+\theta,\ -4\pi+\theta,\ -8\pi+\theta, \dots$ $\Big\}\ x + 2k\pi$

$k = 0, \pm 1, \pm 2, \dots$

So all solutions to $\cos x = \frac{\sqrt{2}}{2}$ are:

$$\boxed{\begin{aligned} x &= \frac{\pi}{4} + 2k\pi, \quad k = 0, \pm 1, \pm 2, \dots \\ x &= \frac{7\pi}{4} + 2k\pi, \quad k = 0, \pm 1, \pm 2, \pm 3, \dots \end{aligned}}$$

In general $\cos x = c$ has two solutions on $[0, 2\pi)$ if $-1 < c < 1$,

$\boxed{\cos x = 1}$ has one solution in $[0, 2\pi)$

namely $x = 0$

all ~~soln~~ solutions $\boxed{x = 2k\pi, \quad k = 0, \pm 1, \pm 2, \dots}$

$\cos x = -1$ has only one solution in $[0, 2\pi)$

namely $x = \pi$

$\left(\begin{array}{l} \theta, \ 2\pi - \theta \\ \theta = \pi \quad \pi, \ \underbrace{2\pi - \pi}_{\pi} \\ \text{a double solution} \end{array} \right)$

all solutions

$x = \pi + 2k\pi$

$= \boxed{(2k+1)\pi \quad k = 0, \pm 1, \pm 2, \dots}$

all odd multiples of π.

Ex. Find a) all solutions in $[0, 2\pi)$ $\quad \theta, \ 2\pi - \theta$

b) all real solutions

c) all solutions in $[-\pi, \pi)$ $\quad \theta, -\theta$.

$$\boxed{\cos x = \frac{1}{2}}$$

Ans a) $x = \cos^{-1}\frac{1}{2} = \boxed{\frac{\pi}{3}}, \quad x = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$ $\qquad \mathbb{Z}$ set of integers

b) $\boxed{\begin{array}{l} x = \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z} \\ x = \frac{5\pi}{3} + 2k\pi, \quad k \in \mathbb{Z} \end{array}}$ $(k = 0, \pm 1, \pm 2, \dots)$ integer

$k = -1$

c) $x = \frac{\pi}{3}, \quad \underbrace{x = -\frac{\pi}{3}}$

$x = -2\pi + \frac{5\pi}{3}$

Ex. Find all solutions in $[0, 2\pi)$

$$\cos x = \frac{1}{5}$$

Ans $x = \cos^{-1}\frac{1}{5}$, $x = 2\pi - \cos^{-1}\frac{1}{5}$

Do the hmwk In notes for trig equations

— o —

$$\cos x = c$$

If $|c| > 1$ no solutions

$|c| = 1$ $c = 1$ $x = 2k\pi,\ k \in \mathbb{Z}$

$c = -1$ $x = (2k+1)\pi,\ k \in \mathbb{Z}$

$|c| < 1$ $x = \cos^{-1} c$, $x = 2\pi - \cos^{-1}(c)$

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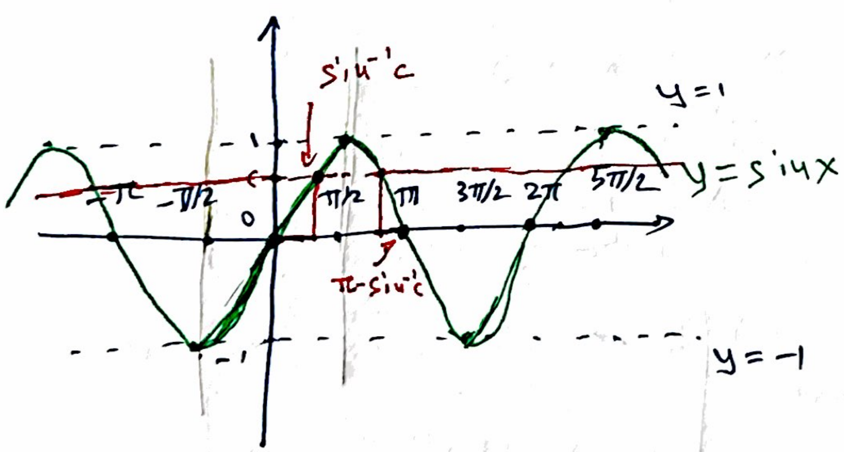

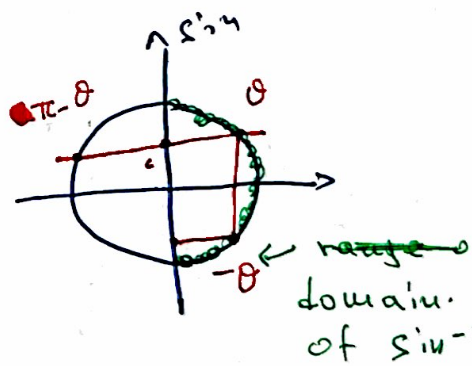

Ex Solve $\sin x = -\frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\frac{\sqrt{3}}{2} = \pi/3$

$\Rightarrow \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

Sol. $x = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) \Leftrightarrow x = \boxed{-\frac{\pi}{3}}$

$x = \pi - \left(-\frac{\pi}{3}\right)$

$= \pi + \pi/3 = \boxed{4\pi/3}$

$$\boxed{\begin{aligned} x &= 2k\pi - \frac{\pi}{3}, \quad k \in \mathbb{Z} \\ x &= 2k\pi + \frac{4\pi}{3}, \quad k \in \mathbb{Z} \end{aligned}}$$