December 4

YESTERDAY

Graph $y = A\sin(\omega x \mp \phi)$

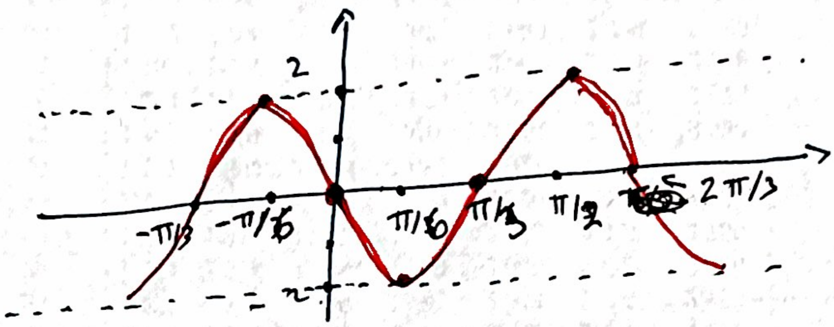
$y = A\cos(\omega x - \phi)$

amplitute $\Rightarrow$ max 2, min -2

Ex 1 Graph $y = 2\cos\left(3x + \frac{\pi}{2}\right)$

amplitute

frequency



Period $P = \frac{2\pi}{3}$

phase shift $3x + \frac{\pi}{2} = 0$ $\boxed{\theta = -\pi/6}$

$\Leftrightarrow 3x = -\frac{\pi}{2}$

$\Leftrightarrow \boxed{x = -\pi/6}$

x	cos	-cos	sin	-sin
θ	A	-A	0	0
$\theta + P/4$	0	0	A	-A
$\theta + P/2$	-A	A	0	0
$\theta + \frac{3P}{4}$	0	0	-A	A
$\theta + P$	A	-A	0	0

$\frac{P}{4} = \frac{1}{4} \cdot \frac{2\pi}{3} = \frac{\pi}{6}$

$-\frac{\pi}{6}$, $-\pi/6 + \pi/6 = 0$

$0 + \pi/6 = \pi/6$,

$\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

$\frac{\pi}{3} + \frac{\pi}{6} = \frac{2\pi + \pi}{6} = \frac{\pi}{2}$

$\pi/2 + \pi/6 = \frac{3\pi + \pi}{6} = 2\pi/3$

x	y
$-\pi/6$	2
0	0
$\pi/6$	-2
$\pi/3$	0
$\pi/2$	2

Graph of $y = -2\sin(3x)$ is the same

$\cos\left(3x + \frac{\pi}{2}\right) = -\sin(3x)$

Verify this

<u>Actually</u> $\cos\left(t+\frac{\pi}{2}\right) = -\sin t$

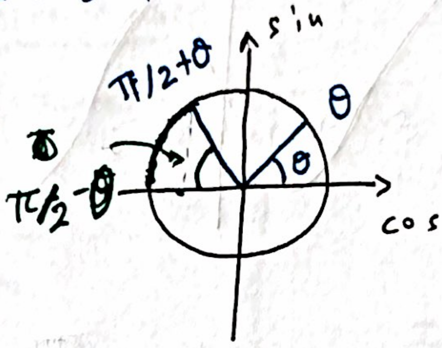
<u>Pf</u> Use addittion formula for cosine

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos\left(t+\frac{\pi}{2}\right) = \cos t \cancelto{0}{\cos\frac{\pi}{2}} - \sin t \cancelto{1}{\sin\frac{\pi}{2}}$$

$$= -\sin t$$

<u>Second way</u>



$$\cos\left(\frac{\pi}{2}+\theta\right) = -\cos(\pi/2-\theta)$$
$$= -\sin\theta$$

$$\boxed{\frac{\pi}{2}+\theta = \pi - (\pi/2-\theta)}$$

— o —

<u>Ex.</u>

Prove $\sin x \overset{\frac{\sin x}{\cos x}}{\tan x} + \cos x = \underbrace{\sec x}_{1/\cos x}$

<u>Sol.</u> Express everything in terms of sine and cosine:

$$\sin x \frac{\sin x}{\cos x} + \cos x = \frac{1}{\cos x}.$$

Start with LHS.

$$LHS = \frac{\sin^2 x}{\cos x} + \cos x$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x}.$$

Verify the identity

$$\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2 + 2\tan^2 x.$$

$1 + \tan^2 x = \sec^2 x$

Sol.

$$LHS = \frac{1}{1+\sin x} + \frac{1}{1-\sin x}$$

$$= \frac{(1-\sin x)+(1+\sin x)}{(1+\sin x)(1-\sin x)}$$

$$= \frac{2}{1-\sin^2 x}$$

$$= \frac{2}{\cos^2 x}$$

$$RHS = 2 + 2\tan^2 x$$

$$= 2(1+\tan^2 x)$$

$$= 2\sec^2 x$$

$$= 2 \cdot \frac{1}{\cos^2 x}$$

$$= \frac{2}{\cos^2 x}$$

Therefore LHS = RHS

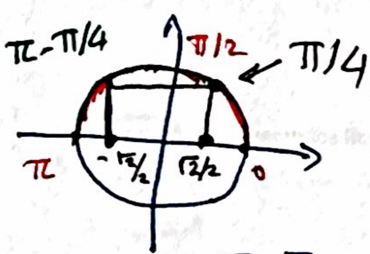
—— o ——

Calculate

a) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 3\pi/4$

$\frac{\sqrt{2}}{2} \rightarrow \text{II}$



$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

c) $\cos\left(\sin^{-1}\left(\frac{3\sqrt{11}}{10}\right)\right) = \frac{1}{10}$

Let $\theta = \sin^{-1}\frac{3\sqrt{11}}{10}$

then $\sin\theta = 3\sqrt{11}/10$

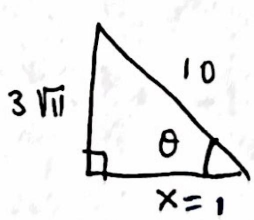
$0 < \theta < \pi/2$

$\boxed{d)\ \sin^{-1}\left(\sin\frac{2\pi}{3}\right)}$

$x^2 + (3\sqrt{11})^2 = 10^2$

$x^2 + 99 = 100$

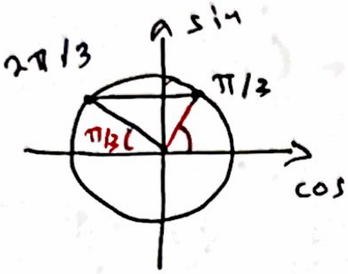
$x^2 = 1 \Rightarrow x = 1$



$\cos\theta = \frac{1}{10}$

d) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ Find an arc θ in $[-\pi/2, \pi/2]$

II s.t. $\sin\theta = \sin\frac{2\pi}{3}$



Therefore

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{\pi}{3}$$

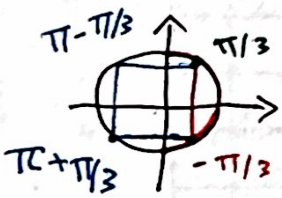
— o —

Give all solutions in $[0, 2\pi)$

a) $\sin x = -\frac{\sqrt{3}}{2}$ b) $\cos x = \frac{1}{2}$

c) $4\cos^2 x - 3 = 0$ d) $2\sin^2 x + \sin x - 1 = 0$

Ans a) I know that $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \Rightarrow \boxed{\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}}$



$$x = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$$

$$x = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$$

b) $\cos x = \frac{1}{2}$ $\boxed{x = \frac{\pi}{3}}$, $x = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

c) Let $u = \cos x$. $4u^2 - 3 = 0 \Leftrightarrow u = \pm\sqrt{\frac{3}{4}}$

$\Leftrightarrow u = \pm\frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2} \Rightarrow \ldots$

$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow \ldots$

$D = 1 - 4\cdot 2\cdot(-1)$
$= 9$

d) Let $u = \sin x$ $2u^2 + u - 1 = 0$ solve $u = \frac{-1 \pm \sqrt{9}}{4} = \begin{cases} 1/2 \\ -1 \end{cases}$

$\sin x = 1/2$
$\ldots$
$\sin x = -1$

Extra Practice [in addition to HW]

1) Find all trigonometric numbers of

a) $-\frac{5\pi}{3}$ b) 480° c) $\frac{7\pi}{6}$

2) Graph

a) $y = 3\sin\left(\frac{x}{2} - \pi\right)$ b) $y = -\frac{1}{2}\cos\left(3x - \frac{\pi}{2}\right)$

3) Angle α is in I and $\sin\alpha = \sqrt{7}/4$

Angle β is in III and $\cos\beta = -\frac{3}{5}$

Find a) $\cos(\alpha+\beta)$ b) $\cos(\alpha-\beta)$

c) $\sin(\alpha+\beta)$ d) $\sin(\alpha-\beta)$

4) Calculate $\tan\left(\sin^{-1}\left(\frac{\sqrt{5}}{3}\right) - \cos^{-1}\left(\frac{3}{4}\right)\right)$

Give exact answer.

5) Find all solutions in $[0, 2\pi)$

$$\sin 2x = \sin x$$

Hint: $\sin 2x = 2\sin x \cos x$