December 11

Last time graphing polynomial functions.

Rational functions

Example Sketch a graph of $f(x) = \dfrac{x^4 - 5x^2 + 4}{x^2 - x - 12}$

Sol. Factor numerator and denominator.

$$x^4 - 5x^2 + 4 = (x^2-1)(x^2-4) = (x-1)(x+1)(x-2)(x+2)$$

$(x^2)^2 - 5(x^2) + 4$

$u^2 + 5u + 4 = (u-1)(u-4)$

$x^2 - x - 12 = (x-4)(x+3)$

$$f(x) = \frac{\overset{+}{(x-1)}\overset{-}{(x-2)}\overset{-}{(x+1)}\overset{-}{(x+2)}}{\underset{+}{(x-4)}\underset{+}{(x+3)}}$$

Find domain $x \neq 4,\ x \neq -3$

$-\infty \quad -3 \quad 4 \quad \infty$

$(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$ V. A. $x = -3$, $x = 4$

Find intercepts x-intercepts $x = 1,\ x = 2,\ x = -1,\ x = -2$

y-intercept $y = f(0) = \dfrac{4}{-12} = -\dfrac{1}{3}$

End point behaviour

$x \to \pm\infty$

$x \to -3$ $\quad x \to -3^-$ $\Rightarrow y \sim \dfrac{+}{0^+} = \infty$

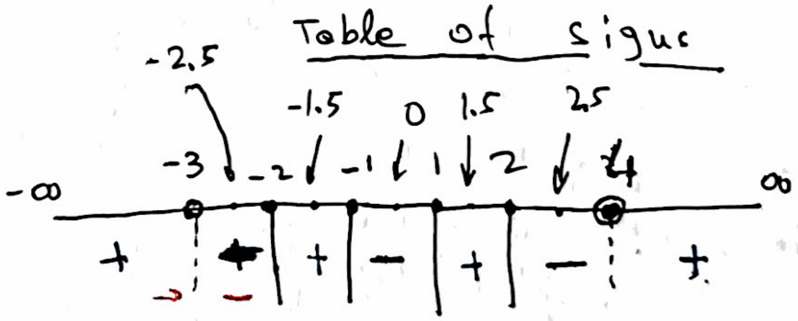
$x \to -3^+ \Rightarrow y \to -\infty$

$x \to 4$

$x \to 4^- \Rightarrow y \to -\infty$

$x \to 4^+ \Rightarrow y \to +\infty$

Table of signs

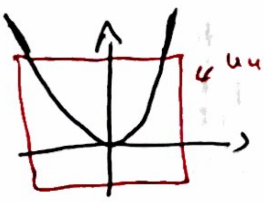


Check

$$f(-2.5) = \frac{(-)\cdot(-)\cdot(-)\cdot(-)}{(-)\,(+)} = (-)$$

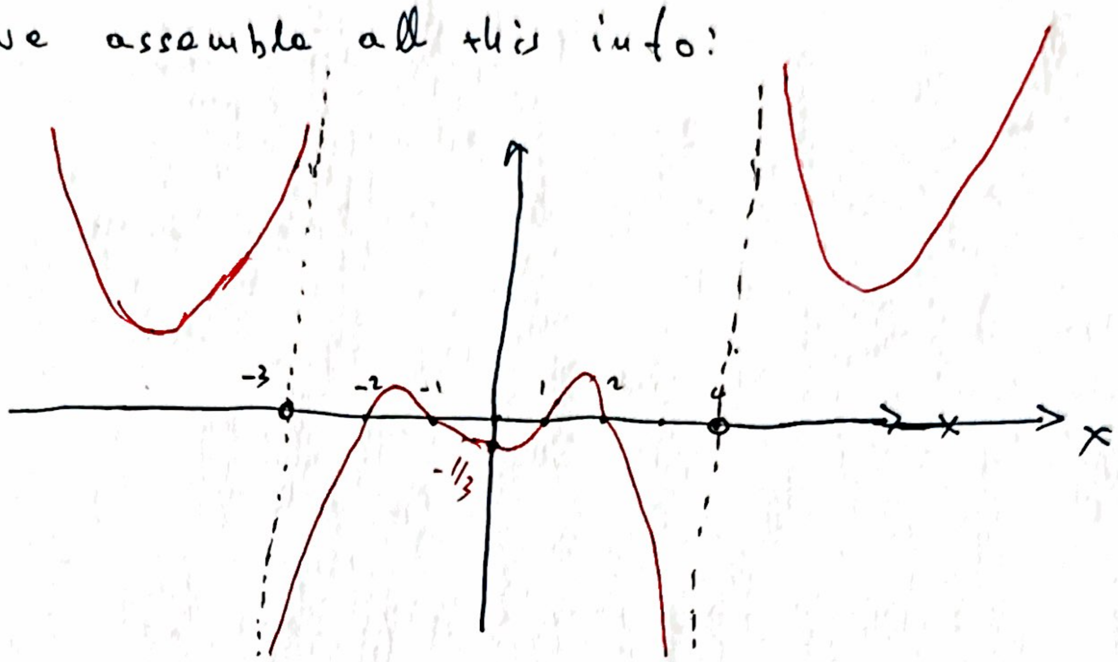
Behaviour at ∞

$x \to \pm\infty \quad f(x) \sim \dfrac{x^4}{x^2} = x^2$



NO H. A.

Now we assemble all this info:



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Review of exponentials and logs.

① Simplify: a) $\cancel{\log_2} \cancel{2}^{52} - 2^{\log_2 14} + \log_2 \underbrace{16}_{2^4} = 52 - 14 + 4$

$= \boxed{42}$

b) $\underset{\log_{10}}{\log}\ \underline{0.001} = \log_{10} 10^{-3} = \boxed{-3}$

$0.001 = 10^{-3}$

c) $\log_{42} 1 = \boxed{0}$

② Verify that $f(x) = \log_2(2x+5) - 3$
$g(x) = \dfrac{2^{x+3} - 5}{2}$
is a pair of inverse functions.

Find domain range of each.

Solution Need to verify $f(g(x)) = x$, $g(f(x)) = x$

$f(g(x)) = \log_2\left(\cancel{2}\left(\dfrac{2^{x+3} - 5}{\cancel{2}}\right) + 5\right) - 3$

$= \log_2(2^{x+3} - \cancel{5} + \cancel{5}) - 3$

$= \cancel{\log_2}(\cancel{2}^{x+3}) - 3$

$= x + \cancel{3} - \cancel{3}$

$= \boxed{x}$ ✓

$g(f(x)) = \dfrac{2^{(\log_2(2x+5) - \cancel{3}) + \cancel{3}} - 5}{2}$

$= \dfrac{\cancel{2}^{\cancel{\log_2}(2x+5)} - 5}{2}$

$= \dfrac{\cancel{2}x + \cancel{5} - \cancel{5}}{\cancel{2}} = \boxed{x}$ ✓

$f(x) = \log_2 \underbrace{(2x+5)}_{\text{needs to be positive}} - 3$

<u>Domain of f</u> Need $2x+5>0 \iff 2x > -5 \iff \boxed{x > -5/2}$

$\left(-\frac{5}{2}, \infty\right)$

<u>Domain of g</u> $(-\infty, \infty)$

$(g = f^{-1})$

<u>Range of f</u> = domain of g = $(-\infty, \infty)$

<u>Range of g</u> = domain of f = $(-5/2, \infty)$

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<u>Ex.</u> Find the domain: a) $f(x) = \ln(x^2 - 2x + 5)$ ($\log_e$)

b) $f(x) = \log_{42}\left(\frac{x-2}{x+1}\right)$

<u>Sol.</u> a) Need $x^2 - 2x + 5 > 0$. This is true allways

so domain $(-\infty, \infty)$.

$D = 4 - 4 \cdot 1 \cdot 5$
$= \boxed{-16}$

No real roots.

$f(x) = x^2 - 2x + 5$

doesn't change sign.

$f(0) = 5$

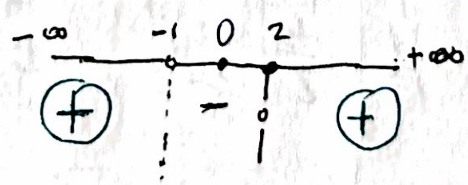
$f(x) > 0$

b) Need $\frac{x-2}{\boxed{x+1}} > 0$

and $x+1 \neq 0$

$\frac{x-2}{x+1}$

$g(x)$ $\quad g(0) = -2$



Domain

$(-\infty, -1) \cup (2, \infty)$

As $x \to \pm\infty$ $y \to \frac{x}{x} = 1$ $\quad y = 1$ H.A.

~~Example~~ Expand $\log_2 \sqrt{\frac{x^3y^5}{8z}}$

$\log xy = \log x + \log y$
$\log \frac{x}{y} = \log x - \log y$
$\log x^n = n \log x$

$\xrightarrow{\text{Expand}}$

Ans $\log_2 \left(\frac{x^3y^5}{8z}\right)^{1/2} = \frac{1}{2} \log_2 \frac{x^3y^5}{8z}$

$= \frac{1}{2}\left(\log_2 (x^3y^5) - \log_2 (8z)\right)$

$= \frac{1}{2}\left(\log_2 x^3 + \log_2 y^5 - \log_2 \overset{2^3}{8} - \log_2 z\right)$

$= \frac{1}{2}\left(3\log_2 x + 5\log_2 y - 3 - \log_2 z\right)$

$\boxed{= \frac{3}{2}\log_2 x + 5\log_2 y - \frac{1}{2}\log_2 z - \frac{3}{2}}$

Solve $\log_3 x + \log_3 (x^2+11x+15) = 3$

Ans Contract LHS $\log_3 (x(x^2+11x+15)) = 3$

Take $\exp_3$ $\cancel{3^{\log_3}}(x^3+11x^2+15x) = 3^3$

$\iff x^3+11x^2+15x = 27$

$\boxed{\iff x^3+11x^2+15x - 27 = 0}$

Do this