MTH 42, Fall 2024

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Third Set of homework

1. Let S_1 and S_2 be two subsets of $\mathbb{R}^n$ with $S_1 \subseteq S_2$. Prove
   (a) If S_1 is spanning then S_2 is also spanning.
   (b) If S_1 is linearly dependent then S_2 is also linearly dependent.
   (c) If S_2 is linearly independent then S_1 is also linearly independent.

2. Decide whether each of the following subsets are vector subspace of the given standard real vector space.
   (a) $\{(x, 3x + y, 0, y - z) : x, y, z \in \mathbb{R}\} \subseteq \mathbb{R}^4$.
   (b) $\{(x, y, z) \in \mathbb{R}^3 : x, y, z \in \mathbb{R}$ and $3\,x - 4\,y = 11\,z\}$.
   (c) The set of points in $\mathbb{R}^2$ that lie in the parabola $y = x^2$.
   (d) The set of points in $\mathbb{R}^3$ that lie in the plane with equation $2x - 3y + 4z = 0$.
   (e) The set of points in $\mathbb{R}^3$ that lie in the plane with equation $2x - 3y + 4z = 8$.
   (f) $\{(x, 2, 3x + 4y, y - z) : x, y, z \in \mathbb{R}\} \subseteq \mathbb{R}^4$.
   (g) $\{(3\,w, 2\,z - 5\,t, x - 4\,y + 5\,t, -2x + z - 3t + 4w) : x, y, z, w, t \in \mathbb{R}\} \subseteq \mathbb{R}^4$.

3. Prove that
$$\{(1, 1, 1, 1, 1), (0, 1, 1, 1, 1), (0, 0, 1, 1, 1), (0, 0, 0, 1, 1), (0, 0, 0, 0, 1)\}$$
   form a basis of $\mathbb{R}^5$.

4. Express each of the vectors of the standard basis of $\mathbb{R}^5$ as a linear combination of the elements of the basis in the previous Question.

5. Which of the following subsets of $\mathbb{R}^3$ are a basis?
   (a) $\{(1, 2, 3), (3, 2, 1)\}$.
   (b) $\{(1, 1, 2), (1, -2, 0), (2, 0, 1)\}$
   (c) $\{(1, 2, 3), (3, 1, 2), (2, 3, 1)\}$.
   (d) $\{(1, 2, 3), (1, 1, 0), (0, 3, 1), (1, 0, 0)\}$.

6. For those subsets in Question 2 that are subspaces find a basis and the dimension.