Quiz 6

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Name: KEY

1. Calculate

$105° = 60° + 45°$

(a) cos 105°.

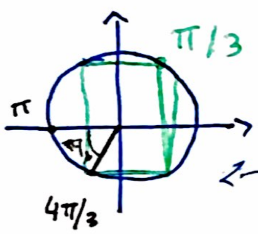
$$\cos(105°) = (\cos 60°)(\cos 45°) - \sin(60°)\cdot\sin(45°)$$

$$= \frac{1}{2}\,\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

(b) $\tan\frac{4\pi}{3}$.

$$\frac{4\pi}{3} = \pi + \pi/3$$



First way

$$\cos\frac{4\pi}{3} = -\cos\pi/3 = \frac{-1}{2}$$

$$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{4\pi}{3}\right) = \frac{-\sqrt{3}/2}{-1/2} = \boxed{\sqrt{3}}$$

Second way

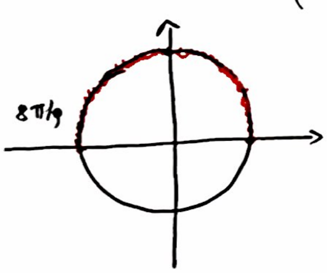
$$\tan\left(\frac{4\pi}{3}\right) = \frac{\cancel{\tan\pi}^{\,0} + \tan\frac{\pi}{3}}{1 - (\cancel{\tan\pi}_{\,0})(\tan\pi/3)}$$

$$= \tan\pi/3$$

$$= \boxed{\sqrt{3}}$$

(c) $\cos^{-1}\left(\cos\frac{8\pi}{9}\right)$.

$$8\pi/9 = \pi - \pi/9 \Rightarrow \frac{8\pi}{9} \text{ in II}$$

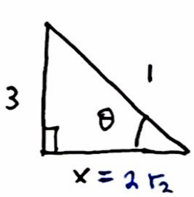


$$\cos^{-1}\left(\cos\frac{8\pi}{9}\right) = \boxed{\frac{8\pi}{9}}$$

(d) $\sin\left(\sin^{-1}\frac{1}{3} - \cos^{-1}\frac{4}{5}\right)$.

Let $\theta = \sin^{-1}\frac{1}{3}$, $\varphi = \cos^{-1}\frac{4}{5}$

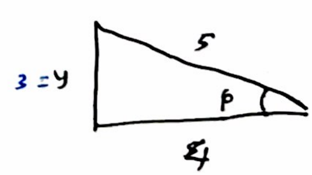
then $\cos\theta = \frac{2\sqrt{2}}{3}$, $\sin\varphi = \frac{3}{5}$



$$\sin(\theta+\varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$$

$$= \frac{1}{3}\,\frac{4}{5} - \frac{2\sqrt{2}}{3}\cdot\frac{3}{5}$$

$$= \boxed{\frac{4-6\sqrt{2}}{15}}$$



$x^2 + 1^2 = 3^2$

$\Rightarrow x = \sqrt{8}$

$y^2 + 4^2 = 5^2$

$\Rightarrow y^2 = 9$

$\Rightarrow y = 3$

2. Verify the identity

$$\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2 + 2\cot^2 x.$$

$LHS = \dfrac{(1-\cos x) + (1+\cos x)}{(1+\cos x)(1-\cos x)}$

$= \dfrac{2}{1-\cos^2 x}$

$= \dfrac{2}{\sin^2 x}$

$RHS = 2(1+\cot^2 x)$

$= 2\csc^2 x$

$= \dfrac{2}{\sin^2 x}$

Therefore $LHS = RHS$.

3. Find all solutions in the interval $[0, 2\pi)$:

$$2\cos^2 x + \cos x - 1 = 0.$$

Let $u = \cos x$

$2u^2 + u - 1 = 0$

$D = 1^2 - 4\cdot 2(-1)$
$= 9$

$u = \dfrac{-1 \pm \sqrt{9}}{4} = \begin{cases} \dfrac{-1+3}{4} = \dfrac{1}{2} \\ \dfrac{-1-3}{4} = -1 \end{cases}$

$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$ or $x = 2\pi - \frac{\pi}{3}$

$\Rightarrow x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

$\cos x = -1 \Rightarrow x = \pi$

$\boxed{x = \frac{\pi}{3} \text{ or } x = \pi, \text{ or } x = \frac{5\pi}{3}}$

4. Graph a full cycle of $y = 3\sin\left(2x - \frac{\pi}{3}\right)$.

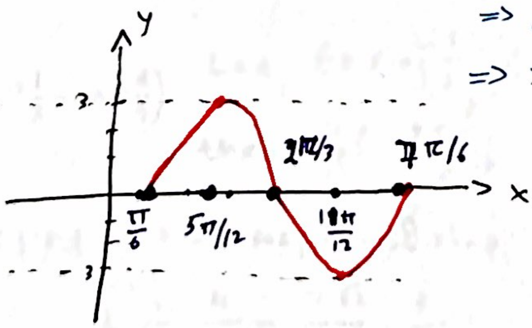
Frequency $\omega = 2$ Period $P = \frac{2\pi}{2} = \pi$

Phase $2x - \frac{\pi}{3} = 0$ amplitude $A = 3$

$\Rightarrow 2x = \frac{\pi}{3}$

$\Rightarrow x = \pi/6$

x	y
$\pi/6$	0
$5\pi/12$	3
$2\pi/3$	0
$11\pi/12$	-3
$7\pi/6$	0



$\frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$

$\frac{5\pi}{12} + \frac{\pi}{4} = \frac{8\pi}{12} = \frac{2\pi}{3}$

$\frac{2\pi}{3} + \frac{\pi}{4} = \frac{11\pi}{12}$

$\frac{11\pi}{12} + \frac{\pi}{4} = \frac{14\pi}{12} = \frac{7\pi}{6}$