BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MTH 30 (Nikos Apostolakis)
Fall 2025

Midterm Exam
1 hour, 25 minutes

Name: KEY

Instructions:

This quiz contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Please print your name clearly on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are allowed to use a calculator.

You are required to show your work on each problem on this quiz. The following rules apply:

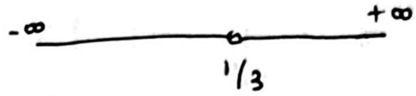
- Make sure to indicate your final answer clearly.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or other work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

1. Let $f(x) = \dfrac{2x}{3x-1}$.

(a) What is the domain of f?

Need $3x - 1 \neq 0 \iff 3x \neq 1$

$\iff \boxed{x \neq 1/3}$



$$\boxed{(-\infty, 1/3) \cup (1/3, \infty)}$$

(b) Show that f is invertible by finding f^{-1}.

We have: $y = f(x) \iff y = \dfrac{2x}{3x-1}$

$$\iff (3x-1)y = 2x$$

$$\iff 3xy - y = 2x$$

$$\iff 3xy - 2x = y$$

$$\iff x(3y-2) = y$$

$$\iff x = \frac{y}{3y-2}$$

Thus f is invertible and

$$\boxed{f^{-1}(y) = \frac{y}{3y-2}}$$

(c) What is the range of f?

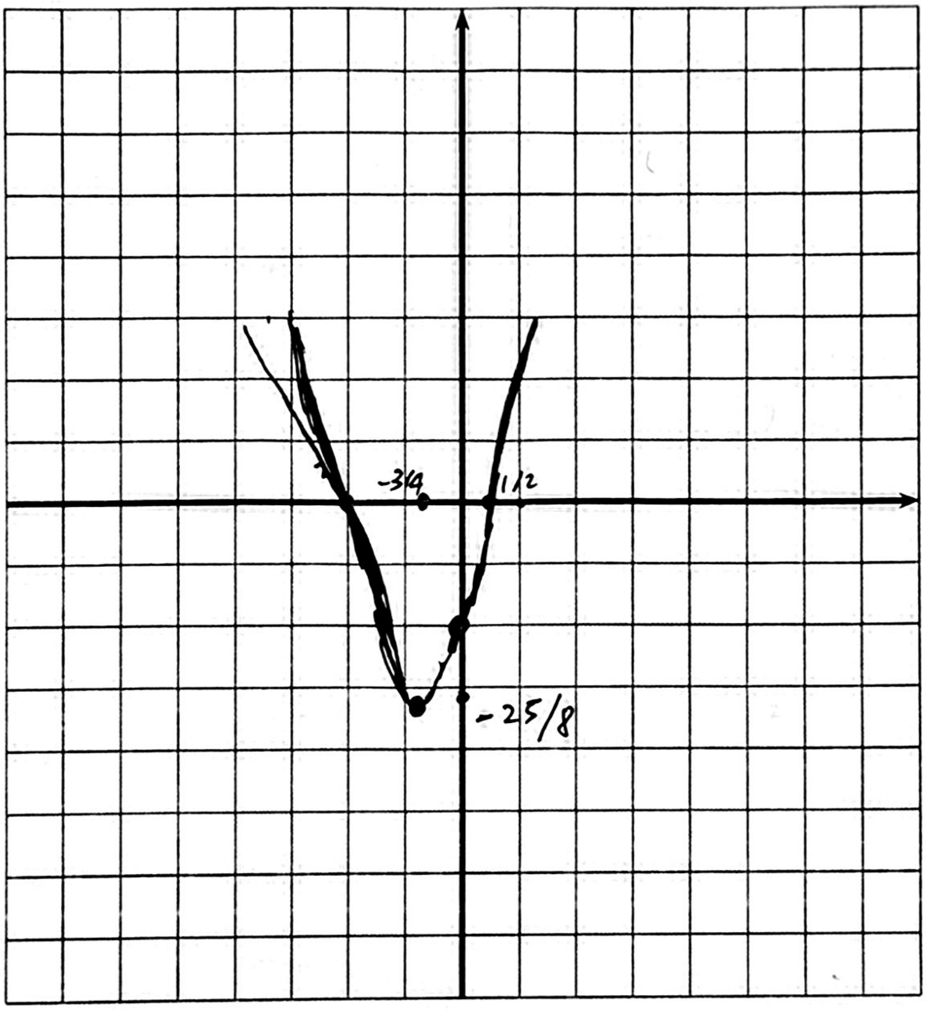
Range of f = Domain f^{-1}

So we have $3y - 2 \neq 0 \iff 3y \neq 2$

$\iff \boxed{y \neq 2/3}$

$$\boxed{(-\infty, 2/3) \cup (2/3, \infty)}$$

2. Graph the function with formula $f(x) = 2x^2 + 3x - 2$ in the grid below. Make sure to indicate the intercepts and the vertex.



$D = b^2 - 4ac$
$= 9 - 4 \cdot 2 \cdot (-2)$
$= 9 + 16$
$= 25$

$h = -\frac{b}{2a} = -\frac{3}{4}$

$k = -\frac{D}{4a} = -\frac{25}{8}$

So Vertex $= \left(-3/4, -25/8\right)$

x-intercepts

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4} = \begin{cases} 2/4 = \boxed{1/2} \\ \frac{-8}{4} = \boxed{-2} \end{cases}$

y-intercept

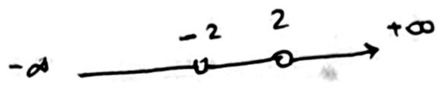
$y = f(0) = \boxed{-2}$

3. Find the domain of each of the following functions:

(a) $f(x) = \dfrac{3x-2}{x^2-4}$.

Need to exclude x's with $x^2 - 4 = 0 \iff x = \pm 2$

Thus domain $x \neq \pm 2$



$$\boxed{(-\infty, -2) \cup (-2, 2) \cup (2, \infty)}$$

(b) $f(x) = \dfrac{3x-2}{x^2+4}$.

Need to exclude x's with $\underline{x^2+4=0}. \iff \underline{x^2 = -4}$

No real solutions

So no values are excluded

$$\boxed{\text{Domain} = (-\infty, \infty)}$$

(c) $f(x) = \sqrt{2x-3}$.

Need $2x - 3 \geq 0 \iff 2x \geq 3$

$\iff \boxed{x \geq 3/2}$

$$\boxed{[3/2, \infty)}$$

4. Solve $x^3 + x^2 - 4x - 4 = 0$.

Candidate rational roots $\pm 1, \pm 2, \pm 4$

Trying $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -4 & -4 \\ & & 1 & 2 & -2 \\ \hline & 1 & 2 & -2 & -6 \end{array}$$

Trying $x = -1$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & \multicolumn{3}{r}{\underbrace{1 \quad 0 \quad -4}_{\text{quotient}}} & \boxed{0} \end{array}$$

$x = -1$ or $x^2 - 4 = 0 \iff \boxed{x = -1 \text{ or } x = -2 \text{ or } x = 2}$

5. Sketch a rough graph for the function with formula

$$f(x) = (x-2)(x+1)^2(2x-3)$$

$1 \quad 1 \quad 2$

$x \quad x^2 \quad 2x$

in the grid below.

Leading term

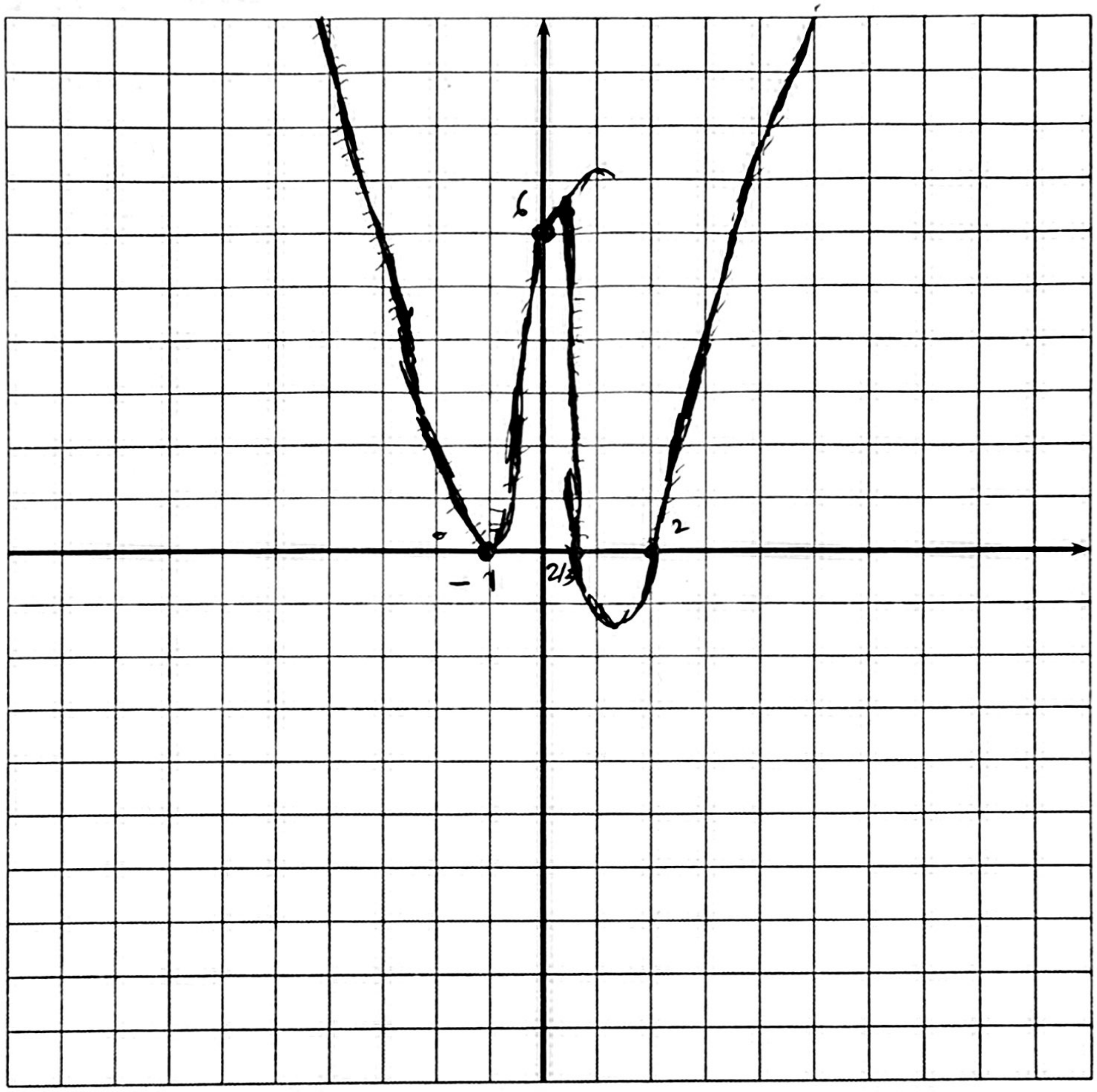
$x \cdot x^2 \cdot 2x = 2x^4$

As $x \to \pm\infty$

$f(x) \sim 2x^4$

y-intercept

$-2 \cdot 1 \cdot (-3) = 6$



x-intercepts

$x - 2 = 0 \iff$ $\underline{x = 2 \quad \text{simple}}$

$(x+1)^2 = 0 \iff$ $\underline{x = -1 \quad \text{double}}$

$2x - 3 = 0 \iff 2x = 3 \iff$ $\underline{x = 3/2 \quad \text{simple}}$

6. Use the graph of the previous question to solve

$$(x-2)(x+1)^2(2x-3) > 0.$$

The intervals whar $y > 0$ in the graph of $y = f(x)$

from previous question

$\boxed{(-\infty, -1) \cup (-1, 2/3) \cup (2, \infty)}$