

September 7

last time 1.4. Composition of functions

$$\underline{f, g} \quad (f \pm g)(x) = f(x) \pm g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ defined when } g(x) \neq 0.$$

$$f(x) = \sqrt{x}, \quad g(x) = x + 3$$

Domain $x \geq 0$

Domain $(-\infty, \infty)$

$$\underline{(f+g)(x) = \sqrt{x} + x + 3,}$$

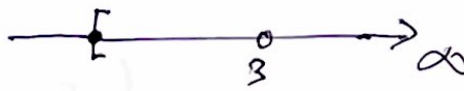
Domain $[0, \infty)$.

$$(f \cdot g)(x) = (\sqrt{x})(x+3) = x\sqrt{x} + 3x.$$

Domain $[0, \infty)$

$$(f - g)(x) = \sqrt{x} - (x+3) = \sqrt{x} - x - 3$$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x+3}$$

Domain $g(x) \neq 0$ so $x \neq -3$ 

Domain $[0, 3) \cup (3, \infty)$

$$\underline{f(x) = \sqrt{x}}, \quad \underline{g(x) = x + 3}$$

$$(f \circ g)(x) = f(g(x)) \qquad (g \circ f)(x) = g(f(x)).$$

$$x \rightarrow \boxed{g} \rightarrow \boxed{f} \rightarrow f(g(x)) \qquad x \rightarrow \boxed{f} \rightarrow \boxed{g} \rightarrow g(f(x))$$

Formula for $f \circ g$

Formula for $g \circ f$

$$(f \circ g)(x) = f(g(x))$$

$$= \sqrt{x+3}$$

$$(g \circ f)(x) = g(f(x))$$

$$= (\sqrt{x}) + 3$$

$$= \sqrt{x} + 3.$$

Domain of $f \circ g$.

$$x \rightarrow \boxed{g} \rightarrow \boxed{f} \rightarrow .$$

- ① x is domain of g
- ② $g(x)$ is in domain of f .

Domain of $g: (-\infty, \infty)$

Domain of $f: [0, \infty)$

$$g(x) \geq 0 \Leftrightarrow x+3 \geq 0 \Leftrightarrow \underline{\underline{x \geq -3}}$$

Domain of $f \circ g$ is $[-3, \infty)$

② $f(x) = \frac{x}{3x-6}$, $g(x) = x+2$.

Find $f \circ g$.

Aus $(f \circ g)(x) = \frac{(x+2)}{3(x+2)-6} = \frac{x+2}{3x+6-6} = \frac{x+2}{\boxed{3x}}$.

Domain $\boxed{f \circ g}$ $g(x)$ $x+2$ is in Domain of f

Domain of f : $3x-6 \neq 0 \Leftrightarrow \underline{\underline{x \neq 2}}$

So I need $\underline{\underline{x+2 \neq 2}} \Rightarrow \underline{\underline{x \neq 0}}$

$(-\infty, 0) \cup (0, \infty)$.

③ $f(x) = \frac{x-2}{3x-6}$, $g(x) = x+2$.

formula: $(f \circ g)(x) = \frac{(x+2)-2}{3(x+2)-6}$

$h(x) = \frac{1}{3}$ $= \frac{x}{3x} = \frac{1}{3}$

Domain $(-\infty, \infty)$ $g(x)$ $= x+2$ is in domain of f .

$\Rightarrow$ Domain $(-\infty, 0) \cup (0, \infty)$

$0 \rightarrow \boxed{g} \rightarrow 2 \rightsquigarrow \boxed{f} \rightsquigarrow \text{X}$

f o g

$$f(g(x)) = \frac{\left(\frac{x-5}{x+3}\right)}{3\left(\frac{x-5}{x+3}\right) + 2}$$

$$= \frac{\frac{x-5}{x+3}}{\frac{3x-15}{x+3} + \frac{2(x+3)}{x+3}}$$

$$= \frac{\frac{x-5}{x+3}}{\frac{2x-15-2x-6}{x+3}} = \frac{x-5}{x-21}$$

$$\hookrightarrow x \neq 21$$

Domain

$\left\{ \begin{array}{l} x \text{ is in Domain of } g \\ g(x) \text{ is in Domain of } f \end{array} \right.$

Dom. g
 $x \neq -3$

Dom of f
 $x \neq \frac{3}{2}$

$$\Leftrightarrow \left\{ \begin{array}{l} x \neq -3 \\ g(x) \neq \frac{3}{2} \Leftrightarrow \frac{x-5}{x+3} \neq \frac{3}{2} \end{array} \right.$$

$$\Leftrightarrow 2(x-5) \neq 3(x+3)$$

$$\Leftrightarrow 2x-10 \neq 3x+9$$

$$\Leftrightarrow -19 \neq x$$

$$x \neq -3$$

$$x \neq -21$$

$$x \neq -19$$

Domain

$$(-\infty, -21) \cup (-21, -19) \cup (-19, -3) \cup (-3, \infty)$$



$$(g \circ f)(x) = g(f(x)) = \frac{\left(\frac{x}{3x-2}\right) - 5}{\left(\frac{x}{3x-2}\right) + 3}$$

$$= \frac{\frac{x - 5(3x-2)}{3x-2}}{\frac{x + 3(3x-2)}{3x-2}}$$

$$= \frac{x - 15x + 10}{x + 9x - 6}$$

$$= \boxed{\frac{-14x + 10}{10x - 6}}$$

Domain $10x - 6 \neq 0 \Rightarrow x \neq \frac{6}{10} = \frac{3}{5}$

$$\boxed{x \neq \frac{3}{5}}$$

dom f

$$\textcircled{x \neq \frac{3}{2}}$$

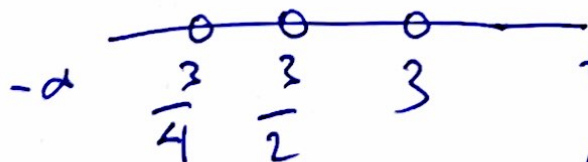
$\left\{ \begin{array}{l} x \text{ in dom } f \\ f(x) \text{ in dom } g \end{array} \right.$

dom g

$$x \neq 3$$

$$\frac{x}{3x-2} \neq 3 \Leftrightarrow x \neq 9x - 6 \Leftrightarrow 8x \neq 6$$

$$\Leftrightarrow x \neq \frac{6}{8} = \frac{3}{4}$$



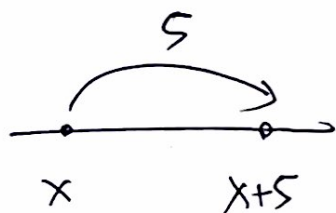
Domain

$$\left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 3\right) \cup (3, \infty)$$

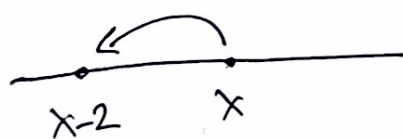
1.5 Transformation of functions

Translation

$$f(x) = x + 5$$



$$f(x) = x - 2$$



Ex $f(x) = x^2$

$$T(x) = x + 3$$

~~f(x)~~

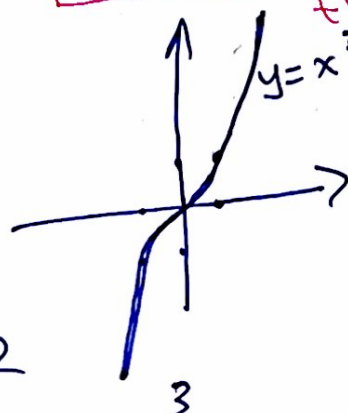
$$(f \circ T)(x) = f(x+3) = (x+3)^2$$

$$(T \circ f)(x) = T(f(x)) = x^2 + 3$$

→ move along the x-axis by -3

→ move along the y-axis by 3

Ex $f(x) = x^3$



graph $g(x) = x^3 - 2$

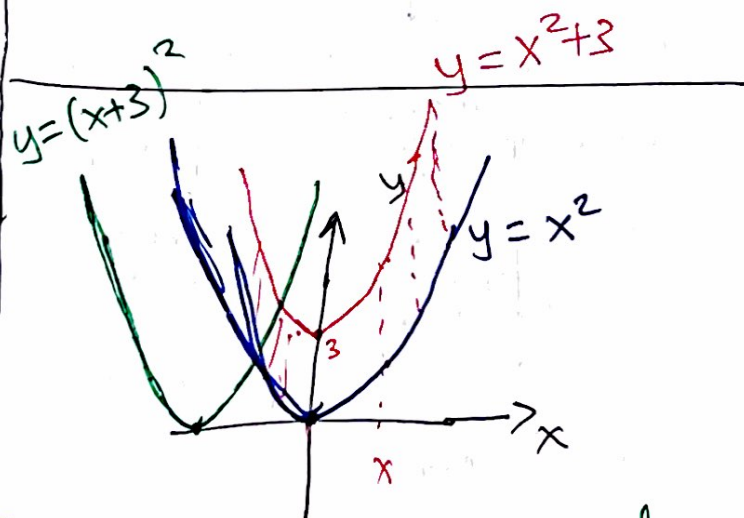
$h(x) = (x-2)^3$

Scaling

$$f(x) = 2x$$

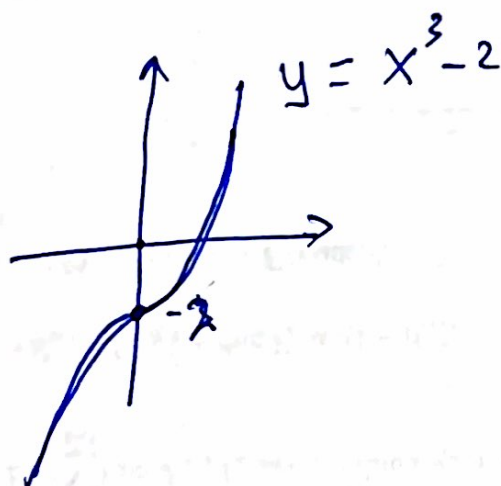
Doubles every value

$$f(x) = \frac{x}{3}$$

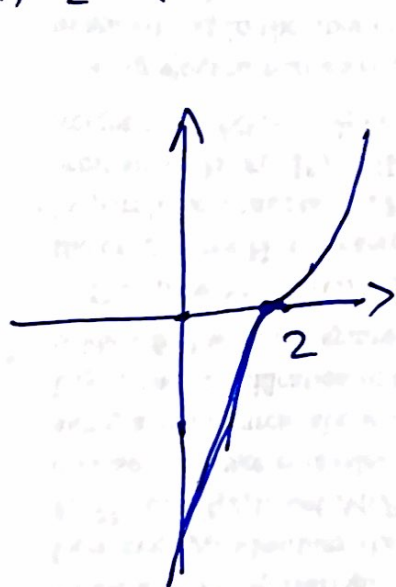


$$g(x) = x^3 - 2$$

$$y = x^3 - 2$$



$$h(x) = (x-2)^3$$



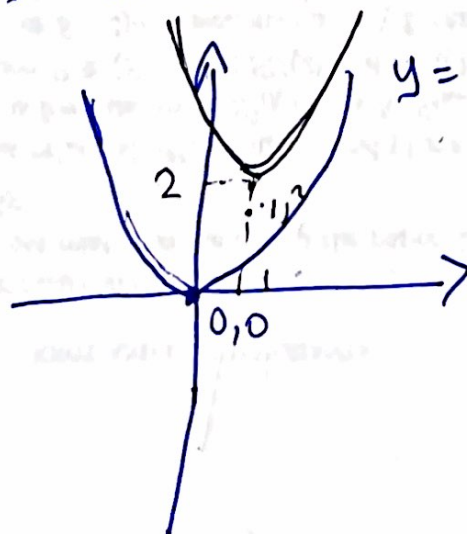
$$y = (x-2)^3$$

Translate the graph of $y = x^3$ 2 units to the right.

right 1 up 2

Example Graph $f(x) = (x-1)^2 + 2$

This function is a translation of $y = x^2$



1 to right

2 up ↑