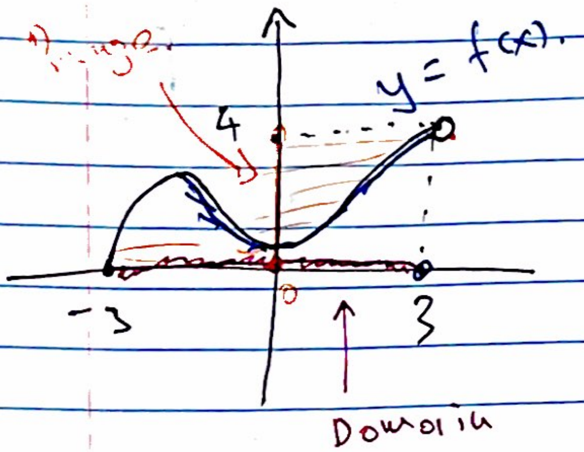
MTH 30 September 5, 2023

Last time — Domain and range (2.2)
Rate of change (2.3)

Today 2.3, 2.4

(1)



(1) Domain of f. [-3, 3)

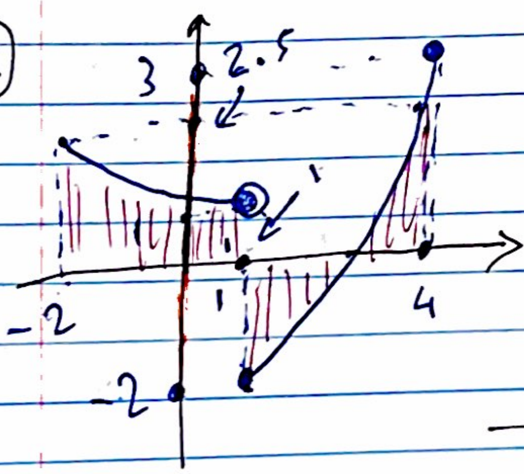
$= \{x : -3 \leq x < 3\}$

(2) Range [0, 4)

f(1) = -2

Piecewise

(2)



Domain. [-2, 4]

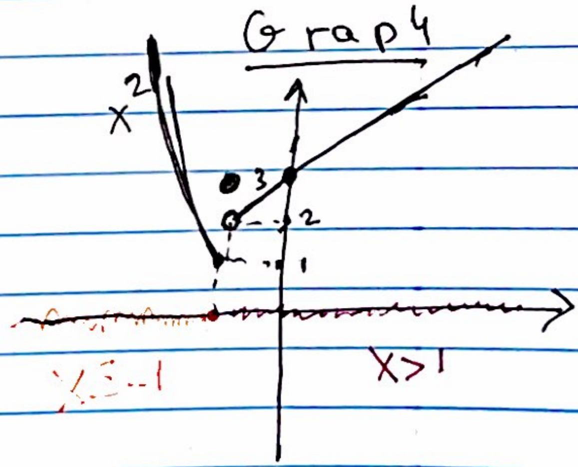
Range [-2, 3]

— o —

(3) Piecewise function

$$f(x) = \begin{cases} x^2, & \boxed{x \leq -1} \\ x+3, & x > -1 \end{cases}$$

Graph

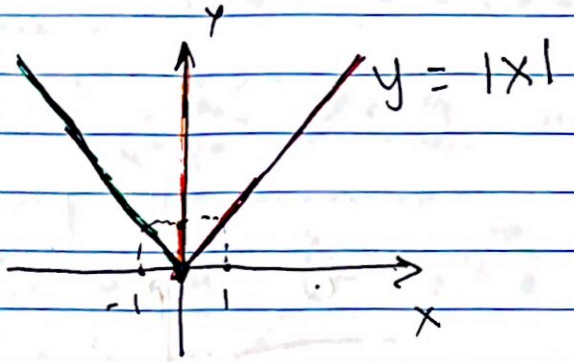


linear function

x	y
-1	2
0	3

④ $f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

$|-3| = -(-3)$
$= 3$



Domain : $\mathbb{R} = (-\infty, \infty)$

Range : $[0, \infty)$

— o —

⑤ $f(x) = \sqrt{x+4}$ Domain?

$x + 4 \geq 0$ ONLY nonnegative numbers have square roots

$\Leftrightarrow \boxed{x \geq -4}$ So domain is $[-4, \infty)$

⑥ $g(x) = \dfrac{3x - 1}{x^2 - 5x + 6}$ Domain?

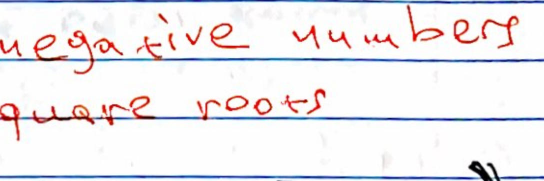
Denominators cannot be 0.

Need to exclude

$x^2 - 5x + 6 = 0$

$\Leftrightarrow (x-2)(x-3) = 0$

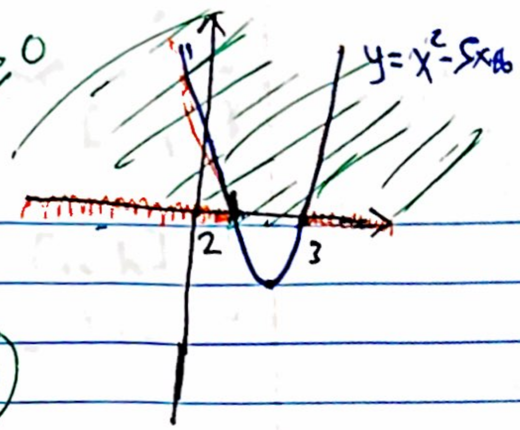
$\Leftrightarrow x = 2, x = 3$



Domain $\{x : x \neq 2, 3\}$

$= (-\infty, 2) \cup (2, 3) \cup (3, \infty)$

7) $f(x) = \sqrt{x^2-5x+6}$ $y \geq 0$



Find domain.

<u>Answer</u> Need $x^2-5x+6 \geq 0$

$\iff x \leq 2$ or $x \geq 3$

Domain $(-\infty, 2] \cup [3, \infty)$.

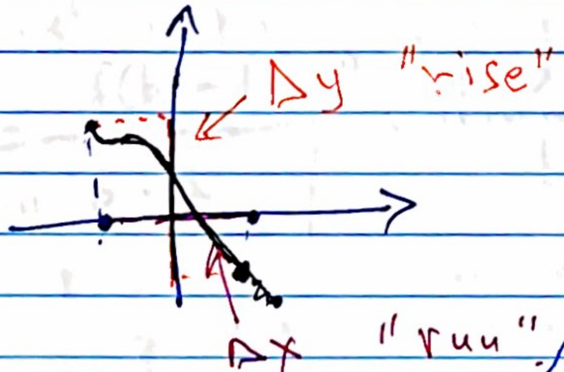
So for domain (so far)

- denominators $\neq 0$
- radicands for even roots ≥ 0
  ↳ what's inside the root

2.3 Average Rates of change

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

x_1, x_2



$\Delta x = 2-(-1)$
$= 3$

$\Delta y = f(2) - f(-1)$
$= 8-(-1) = 9$

Ex. Find $\frac{\Delta y}{\Delta x}$, interval $[-1, 2]$

$f(x) = x^3$

$$\frac{\Delta y}{\Delta x} = \frac{9}{3} = 3$$

If f is increasing ~~then~~ Δ on $[a, b]$

then $\frac{\Delta y}{\Delta x} > 0$ on $[a, b]$

everywhere on

f is decreasing on $[a, b]$ then

$\frac{\Delta f}{\Delta x} < 0$ for any x_1, x_2 on $[a, b]$

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

say $x_2 > x_1$ $\to > 0$

increasing $\frac{+}{+} > 0$

decreasing $\frac{-}{+} < 0$

Example. $f(x) = 3x - 5$

Find average rate of change on $[a, b]$

Solution $\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \frac{3b - 5 - (3a - 5)}{b - a}$

$$= \frac{3b - \cancel{5} - (3a + \cancel{5})}{b - a}$$

$$= \frac{3b - 3a}{b - a} = \frac{3\cancel{(b-a)}}{\cancel{b-a}}$$

$$= \boxed{3}$$

$y = \boxed{3}x - 5$

linear function

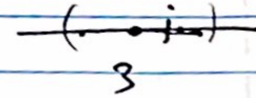
slope 3

increasing

(4) Find rate of change of

$$f(x) = 2x^2 - 3x + 5$$

for $x_1 = 3$, $x_2 = 3+h$



Ans $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$\cancel{f(x_2)}$

$$\begin{aligned} f(x_2) &= f(3+h) \\ &= 2(3+h)^2 - 3(3+h) + 5 \\ &= 2(9+6h+h^2) - 9 - 3h + 5 \\ &= 18 + 12h + 2h^2 - 3h + 4 \\ &= 2h^2 + 15h + \cancel{24} \end{aligned}$$

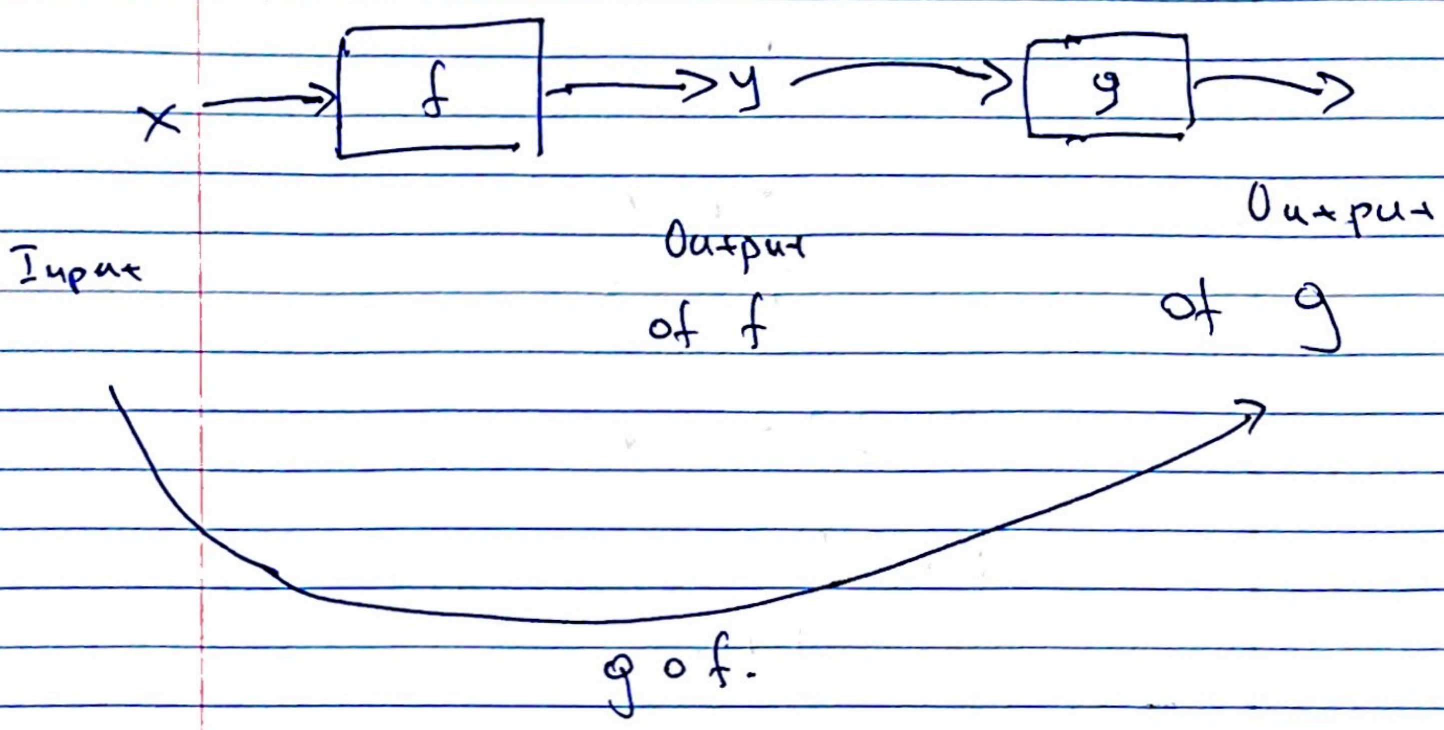
$$f(x_1) = f(3) = 2(3)^2 - 3(3) + 5 = 18 - 9 + 5 = 14$$

$$\begin{aligned} \Delta y = f(x_2) - f(x_1) &= 2h^2 + 15h + \cancel{24} - \cancel{14} \\ &= \boxed{2h^2 + 15h} \end{aligned}$$

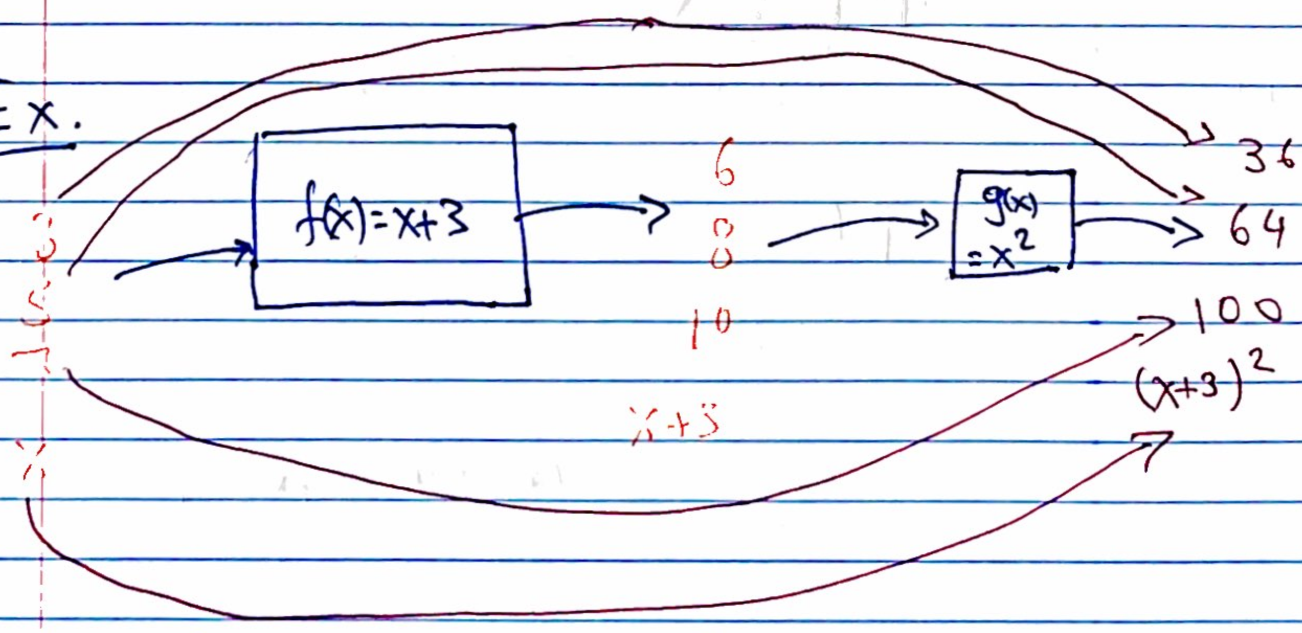
$$\Delta x = x_2 - x_1 = (3+h) - 3 = \boxed{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{2h^2 + 15h}{h} = \frac{\cancel{h}(2h+15)}{\cancel{h}} = \boxed{2h+5}$$

2.4 Composition of functions



Ex.



So $(g \circ f)(x) = (x+3)^2$

I can simplify $(f \circ g)(x) = x^2 + 6x + 9$

$f(x) = x+3 \qquad g(x) = x^2$

$$x \longrightarrow \boxed{g(x)=x^2} \longrightarrow x^2 \longrightarrow \boxed{\begin{matrix} f(x) \\ = x+3 \end{matrix}} \longrightarrow x^2+3$$

$$(f \circ g)(x) = x^2 + 3$$

So $\boxed{f \circ g}$ and $\boxed{g \circ f}$ are usually different functions.

In general:

$$\boxed{(f \circ g)(x) = f(g(x))}$$

$f(\ ) = 3(\ ) - 5$

Example ① $f(x) = 3x - 5$, $g(x) = \sqrt{x-1}$

a) find formula for $f \circ g$.

Ans $(f \circ g)(x) = f(g(x))$

$$= 3\left(\sqrt{x-1}\right) - 5$$

$$= 3\sqrt{x-1} - 5$$

b) Find formula for $g \circ f$

<u>Answer</u> $(g \circ f)(x) = g(f(x))$

$$= \sqrt{(3x-5) + 1}$$

$$= \sqrt{3x - 6}$$