Name: _____ **ANSWERS** _____

> **Directions:** Write your answers in the provided space. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly.

1. (20 points) For the following sample

$$0 \quad 2 \quad 3 \quad 5$$

compute

(a) (5 points) The sample mean $\bar{x}$.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{10}{4} = 2.5$$

First way

x	x^2
0	0
2	4
3	9
5	25
Σ 10	38

$$s^2 = \frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1} = \frac{38 - \frac{100}{4}}{3}$$

$$= \frac{38 - 25}{3}$$

$$= \frac{13}{3} \approx 4.33$$

(b) (15 points) The sample standard deviation s.

$$s = \sqrt{s^2}$$

$$= \sqrt{4.33}$$

$$\approx \boxed{2.03}$$

Second way for $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

x	$x-\bar{x}$	$(x-\bar{x})^2$
0	-2.5	6.25
2	-0.5	0.25
3	0.5	0.25
5	2.5	6.25
Σ		13.0

$$= \frac{13}{4}$$

$$\approx 4.33$$

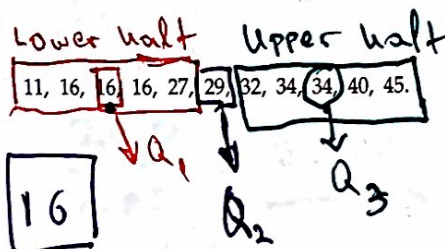$P(R) = 0.40$ $P(W) = 0.60$ $P(W \text{ given } R) = 0.80$

2. (20 points) For the month of April the probability of a day being rainy is 0.40, while the probability of being windy is 0.60. About 80% of the rainy days are also windy. What is the probability that a random day of April is

(a) (10 points) both rainy and windy?

$$P(R \text{ and } W) = P(R) \cdot P(W \text{ given } R)$$
$$= 0.40 \cdot 0.80$$
$$= \boxed{0.32}$$

(b) (10 points) rainy or windy?

$$P(R \text{ or } W) = P(R) + P(W) - P(R \text{ and } W)$$
$$= 0.40 + 0.60 - 0.32$$
$$= 0.68$$

3. (20 points) For the following data:

Lower half   Upper half $n = 11$ $\dfrac{n+1}{2} = 6$

11, 16, 16, 16, 27, 29, 32, 34, 34, 40, 45.

Q_1 Q_2 Q_3

(a) (5 points) Find the mode.

Mode is $\boxed{16}$

(b) (10 points) Find the median, and the first and third quartiles.

There are $n = 11$ values. The median is in position $\dfrac{n+1}{2} = \dfrac{12}{2} = 6$. Thus median is the 6th number.

$$\boxed{\tilde{X} = 29}$$

$$\boxed{Q_1 = 16}$$
$$\boxed{Q_3 = 34}$$

(c) (5 points) What is the percentile rank of 16?

There are 4 out of 11

$X \leq 16$

$\dfrac{4}{11} \approx 0.3636\ldots$ $\boxed{37 \text{ percentile}}$

4. (20 points) In a stack of selves I have 500 books. They are either in English or in Greek, and their content is either Math, Non Fiction other than math, or Fiction. The number of the books according their language and their category is shown in the table below

	Math	Non-Fiction	Fiction	Total
English	150	100	30	280
Greek	100	50	70	220
Total	250	150	100	500

If I pick up a book at random from that stack of selves what is the probability that the book is

(a) (5 points) A Greek book.

$$P(G) = \frac{220}{500} = 0.44$$
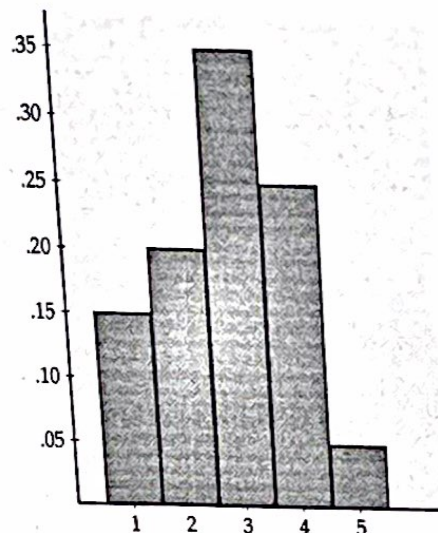
(b) (5 points) A Math book.

$$P(M) = \frac{250}{500} = 0.50$$

(c) (5 points) A Greek book or a Math book.

$$P(M \text{ or } G) = \frac{250 + 220 - 100}{500} = \frac{370}{500}$$
$$= 0.74$$

(d) (5 points) Given that the book was Greek, what's the probability that it was a Math book?

$$P(M \text{ given } G) = \frac{100}{220} \approx 0.45$$

5. (20 points) The histogram of the probability distribution of a random variable X is shown below.

(a) (7 points) Find the expected value $E(X)$.

x	P(x)	x P(x)
1	0.15	0.15
2	0.20	0.40
3	0.35	1.05
4	0.25	1.00
5	0.05	0.25
		2.85

$$E(X) = \sum x \, p(x) \; (3.75)$$

$$= 2.85$$

(b) (13 points) Find the standard deviation σ_X.

x	P(x)	x²	x² P(x)
1	0.15	1	0.15
2	0.20	4	0.80
3	0.35	9	3.15
4	0.25	16	4.00
5	0.05	25	1.25
			9.35

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$

$$= 9.35 - 3.85^2$$

$$= 9.35 - 8.1225$$

$$= 1.2275$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.2275} \approx 1.11$$