BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 23 — Exam 2
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Name: ANSWERS

Directions: Write your answers in the provided space. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly.

1. 75% of the residents of Pleasantville like banana splits. If we randomly select 20 people from Pleasantville:
   (a) How many of those selected we expect to like banana splits?

X = number of people that like banana split.

X binomial $n=20$, $p=0.75$, $q=0.25$

$$E(X) = n \cdot p = 20 \times 0.75 = \boxed{15}$$

We expect $\boxed{15}$ people to like banana split.

   (b) What is the standard deviation of the number of those selected that like banana splits?

$$\sigma_X = \sqrt{n \cdot p \cdot q} = \sqrt{15 \times 0.25} = \sqrt{3.75} \approx 1.94$$

   (c) What is the probability that exactly 15 of the selected people like banana splits?

$$P(X=15) = \boxed{0.2023}$$

   (d) What is the probability that more than 13 but at most 18 of the selected people like banana splits?

$$P(13 < X \leq 18) = P(X \leq 18) - P(X \leq 13) = 0.9757 - 0.2142 = \boxed{0.7615}$$

2. Let X be a random variable that represents the length of time it takes a student to complete an exam. It was found that x has an approximately normal distribution with mean $\mu = 2.4$ hours and standard deviation $\sigma = 0.8$ hours.

(a) What is the probability that a randomly selected student finishes the exam within the allocated time of 3 hours?

$$X = 3 \Rightarrow Z = \frac{3-2.4}{0.8} = \frac{0.6}{0.8} = 0.75$$

$$P(X \leq 3) = P(Z \leq 0.75)$$

$$\boxed{= 0.77337}$$

(b) Suppose 25 students are selected at random. What is the probability that $\bar{x}$, the mean time of completing the exam for these 25 students, is not more than 2 hours?

Since X is n.d.

$\bar{X}$ is n.d. with

$\mu_{\bar{X}} = \mu_X = 2.4$

$\sigma_{\bar{X}} = \sigma_X / \sqrt{n} = \frac{0.8}{\sqrt{25}}$

$= 0.16$

$$\bar{X} = 2 \Rightarrow Z = \frac{2-2.4}{0.16} = -2.5$$

$$P(\bar{X} \leq 2) = P(Z \leq -2.5)$$

$$= \boxed{0.00621}$$

4. Colette is self-employed, selling cosmetics at home parties. She wants to estimate the average amount a client spends per year at these parties. A random sample of 16 receipts had a mean of $\bar{x} = \$340.70$ with a standard deviation of $s = \$60.15$. Find a 90% confidence interval for the mean amount μ spent by all clients. Assume x has an approximately normal distribution.

We have a small sample ($n = 16$) drawn from a n.d. population. So we use t-distribution with $r = 16 - 1 = 15$ degrees of freedom, with $c = 0.90$

From the tables we have $t_{0.90} = 1.753$

The error is then

$$E = t_{0.90} \frac{60.15}{\sqrt{16}}$$

$$= 1.753 \frac{60.15}{4}$$

$$= 1.753 \cdot 15.0375$$

$$\approx 26.36$$

So the 90% confidence interval is

$$340.70 - 26.36 \leq \mu \leq 340.70 + 26.36$$

that is

$$\boxed{314.34 \leq \mu \leq 367.06}$$

5. Jorge lives in Pleasantville and hates banana splits. He can't believe that 75% of his fellow residents like that stuff. He decides to test the hypothesis H_0: $p = 0.75$ with alternative hypothesis H_a: $p < 0.75$. In a random sample of 100 residents he finds that 73 like banana splits.

Is this sufficient evidence to reject H_0 at the level of significance $\alpha = 0.05$?

We have $\hat{p} = \frac{73}{100} = 0.73$, and $\hat{q} = \frac{27}{100} = 0.27$.

Since $n \cdot \hat{p} = 73$ and $n \cdot \hat{q} = 27$ both larger than 5 this sample is sufficiently large.

We have $\sigma_{\hat{p}} = \sqrt{\frac{p \cdot q}{n}} = \sqrt{\frac{0.75 \cdot 0.25}{100}} \approx 0.0433$ and so the test statistic is

$$z = \frac{0.73 - 0.75}{0.0433}$$

$$\approx -0.46$$

Two alternative ways of proceeding:

P-value

$P(z \leq -0.46) = 0.32997$

Since p-value $> \alpha = 0.05$ there is not enough evidence to reject H_0

Rejection region

This is a one-tailed test (left-tailed). From the table of t-values for $\nu = \infty$ we have $z_{0.05} = 1.645$

So we have the rejection region

Test statistic is outside rejection region.

$$\boxed{\text{There is not enough evidence to reject } H_0.}$$