BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 23
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Exam 2
December 10, 2025

Name: KEY

Directions: Write your answers in the provided space. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly.

1. The probability distribution of a discrete random variable X is given by:

x	5	10	20	25	Σ
$P(x)$	0.30	0.20	0.35	0.15	
$xp(x)$	1.5	2.0	7.0	3.75	14.24 $= \mu$
x^2	25	100	400	625	
$x^2p(x)$	7.5	20	140	93.75	$\underbrace{261.25}_{\Sigma x^2 p(x)}$

(a) Find the expected value $E(X)$.

$$E(X) = \Sigma\, xp(x) = \boxed{14.24}$$

(b) Find the variance σ^2 of X.

$$\sigma^2 = \Sigma x^2 p(x) - \mu^2$$
$$= 261.25 - (14.24)^2$$
$$= 261.25 - 202.7776$$
$$= \boxed{58.4724}$$

(c) Find the standard deviation σ of X.

$$\sigma = \sqrt{\sigma^2} = \sqrt{58.4724} \approx \boxed{7.6467}$$

2. 75% of the residents of Pleasantville like banana splits. If we randomly select 20 people from Pleasantville:

(a) How many of those selected we expect to like banana splits?

Let X stand for the number of people among the sample that like banana splits. Then X follows Binomial distribution with $n = 20$, $p = 0.75$. Thus the expected value of X is

$$E(X) = n \cdot p$$
$$= 20 \times 0.75$$
$$= \boxed{15.}$$

(b) What is the standard deviation of the number of those selected that like banana splits?

$$\sigma_X = \sqrt{n \cdot p \cdot q}$$

$$q = 1 - 0.75 = 0.25$$

$$= \sqrt{15 \cdot 0.25}$$

$$= \sqrt{3.75} \approx \boxed{1.94}$$

(c) What is the probability that exactly 15 of the selected people like banana splits?

$$P(X = 15) = \boxed{0.2023}$$

From the P.D.F. of binomial distribution tables

(d) What is the probability that more than 13 but at most 18 of the selected people like banana splits?

$$P(13 < X \leq 18) = P(X \leq 18) - P(X \leq 13)$$

From CDF tables of binomial distribution

$$= 0.9757 - 0.2142$$

$$= \boxed{0.7615}$$

3. Let X be a random variable that represents the length of time it takes a student to complete an exam. It was found that X has an approximately normal distribution with mean $\mu = 2.4$ hours and standard deviation $\sigma = 0.8$ hours.

(a) What is the probability that a randomly selected student finishes the exam within the allocated time of 3 hours?

The z-score for $x = 3$ is $z = \dfrac{3 - 2.4}{0.8}$

$$= \frac{0.6}{0.8}$$

$$= 0.75$$

Thus $P(X \le 3) = P(Z \le 0.75)$

$$= \boxed{0.73337}$$

From the tables of left tails of Z.

(b) Suppose 25 students are selected at random. What is the probability that $\bar{x}$, the mean time of completing the exam for these 25 students, is not more than 2 hours?

The sample mean $\bar{X}$ is normally distributed because X is normally distributed.

$\mu_{\bar{X}} = \mu_X = \boxed{2.4}$

$\sigma_{\bar{X}} = \dfrac{\sigma_X}{\sqrt{n}} = \dfrac{0.8}{\sqrt{25}}$

$= \dfrac{0.8}{5}$

$= \boxed{0.16}$

$\bar{X} = 2 \Rightarrow z = \dfrac{2 - 2.4}{0.16}$

$= \dfrac{-0.4}{0.16}$

$= \boxed{-2.5}$

So $P(\bar{X} \le 2) = P(Z \le -2.5)$

$= \boxed{0.00621}$ ← From Z-tables

4. Colette is self-employed, selling cosmetics at home parties. She wants to estimate the average amount a client spends per year at these parties. A random sample of 16 receipts had a mean of $\bar{x} = \$340.70$ with a standard deviation of $s = \$60.15$. Find a 90% confidence interval for the mean amount μ spent by all clients. Assume X has an approximately normal distribution.

We are sampling from a normal distribution and the sample size is $n = 16$. To calculate the standard error we use t-distribution with $\nu = 16 - 1 = 15$ degrees of freedom. From the tables of critical values of the t-distribution we have $t_{90\%} = 1.753$.

Thus standard error is

$$E = t_c \cdot \frac{s}{\sqrt{n}}$$

$$= 1.753 \cdot \frac{60.15}{\sqrt{16}}$$

$$= \frac{105.44295}{4}$$

$$\approx \boxed{26.36}$$

The confidence interval is then

$$\bar{x} - E \le \mu \le \bar{x} + E$$

OR

$$340.70 - 26.36 \le \mu \le 340.70 + 26.36$$

and finally

$$\boxed{314.34 \le \mu \le 367.06}$$

5. Jorge lives in Pleasantville and hates banana splits. He can't believe that 75% of his fellow residents like that stuff. He decides to test the hypothesis H_0: $p = 0.75$ with alternative hypothesis H_a: $p < 0.75$. In a random sample of 100 residents he finds that 73 like banana splits.

Is this sufficient evidence to reject H_0 at the level of significance $\alpha = 0.05$?

$\hat{p} = \frac{73}{100} = 0.73$

$\left.\begin{array}{l} n\hat{p} = 73 \\ n\hat{q} = 100-73 = 27 \end{array}\right\}$ Both greater than 5

So $\hat{p}$ follows approximatel normal distribution with

$\mu_{\hat{p}} = p = \boxed{0.75}$

$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = \sqrt{\frac{0.75 \times 0.25}{100}} = \sqrt{0.00175} \approx \boxed{0.0433}$

We calulate the test-statistic

$\hat{p} = 0.73 \Rightarrow z = \frac{0.73 - 0.75}{0.0433}$

$\approx \boxed{-0.46}$

P-value method

Since Ha is "$p < 0.75$" we have left tailed test. The p-value is then

$P(Z \leq -0.46) = 0.32276$

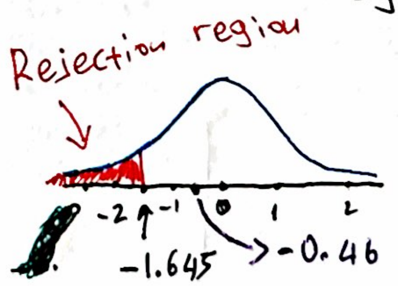
and this is more than our significance level $\alpha = 0.05$.

Thus we don't have enough evidence to reject the null hypothesis.

Rejection region method

From the table of critical values of the t-distribution with ∞ degrees of freedom we have that the critical value that corresponds to $\alpha = 0.05$ (one-tail) is $\boxed{z_{0.05} = 1.645}$

Since we have left-tail test the rejection region is



Since the test statistic is outside the rejection region, we don't have enough evidence to reject H_0.