Review questions for the MTH 23 final

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1. Compute the median, mean, and standard deviation for the following sample:

$$15 \quad 20 \quad 3 \quad 7 \quad 5.$$

2. The probability of rain on any day for the month of April is 25%. The probability that a day in April is windy is 36%, and the probability of both is 11%. What is the probability that a randomly selected day in April will be rainy or windy?

3. The following table relates the weights and heights of a group of individuals participating in an observational study.

	Tall	**Medium**	**Short**	**TOTAL**
Obese	18	28	14	
Normal	20	51	28	
Underweight	12	25	9	
TOTAL				

   (a) Find the total for each row and column.
   (b) Find the probability that a randomly chosen individual from this group is Tall.
   (c) Find the probability that a randomly chosen individual from this group is Tall **given** that the individual is Obese.
   (d) Find the probability that a randomly chosen individual from this group is Obese **or** Tall.
   (e) Find the probability that a randomly chosen individual from this group is **not** Obese.

4. About 25% of students in a college smoke. We randomly select a group of 30 students from that college.
   (a) What is the probability that 3 or more students in that group smoke?
   (b) What is the probability that more than 2 but less than 8 students in that group smoke?
   (c) What is the expected number of smokers in that random sample?
   (d) What is the standard deviation?

5. A company that makes light bulbs claims that its bulbs have a mean life of $\mu = 750$ hours and a standard deviation $\sigma = 20$ hours. Let X be the random variable that counts the life of a randomly selected light bulb made by that company. Assume that X is normally distributed, and that the company's claim is true.

(a) If we select a random light bulb what is the probability that the bulb will last between 745 and 755 hours?

(b) What is the probability that a randomly selected sample of 64 light bulbs will have a mean life $\bar{x}$ between 745 and 755 hours?

6. The college of Question 4 decided to run a campaign aiming at reducing the proportion of smokers among its students. To judge whether the campaign was successful, a random sample of 100 students was selected after the campaign was over, and 16 of them were smokers.

   (a) Calculate the proportion $\hat{p}$ of smokers in this sample.

   (b) If the proportion p of smokers in the whole student population has not changed after the campaign was run, what is the probability that we get the sample proportion of Part (a) or smaller in a random sample of a 100 students?

   (c) Based on your answer in Part (b)), at the significance level of $\alpha = 5\%$ is there enough evidence for the college to conclude that the proportion of smokers among its students has decreased after the campaign was run?

   (d) Repeat the previous part but at the significance level $\alpha = 1\%$.

7. From previous studies it is known that the GPA of students in a university has a standard deviation $\sigma = 0.51$. A random sample of 120 students from that university yields a mean GPA $\bar{x} = 2.71$. Construct a 99% confidence interval for the mean GPA μ of all the students in that university.

8. The weight of artificial sapphires manufactured by a company is assumed to be normally distributed. The company wishes to estimate the mean weight μ of the artificial sapphires it produces. A random sample of $n = 12$ sapphires yields a mean $\bar{x} = 6.75$ carats, and a standard deviation $s = 0.33$ carats. Construct a 90% confidence interval for μ.

9. Construct a 95% confidence interval for the proportion of smokers among the students of the College in Questions 4 and 6, based on the sample in Question 6.

10. The mean breaking distance of a certain automobile car on wet road from 60 miles per hour, was found to be normally distributed with mean $\mu = 159$ feet and standard deviation $\sigma = 23.5$ feet. The manufacturer wants to reduce the breaking distance and installs a new type of tires with a supposedly better grip. In 45 tests with the new tires, the mean breaking distance was found to be $\bar{x} = 148$ feet.

    Is there enough evidence, at the 1% level of significance, to conclude that the actual mean breaking distance is reduced?

11. Let x be the random variable representing the hemoglobin count (HC) in human blood measured in grams per milliliter. In healthy adult females x has a normal distribution with mean $\mu = 14.2$ and standard deviation $\sigma = 2.4$. Suppose that a female patient had 10 blood tests over the past year, and the sample mean of the HC was $\bar{x} = 15.2$. At the level of significance $\alpha = 0.01$ determine whether there is enough evidence to conclude that this patient's HC is higher than the population average.

12. As fallen trees lie on the forest floor they slowly decay. We want to know whether the amount of light present has an impact on the rate of decay. In the following table x is a random variable representing the remaining percentage of the total mass of a log after it was exposed to the elements for three weeks while y represents the number of hours of sunlight the log received each day.

x	.56	.46	.74	.92	.95
y	1.2	2.1	2.9	3	4.2

(a) Plot a scatter diagram of the data. Remember to label your axes appropriately and choose a consistent scale.

(b) (2 points) Based on a scatter diagram, would you estimate the correlation coefficient to be positive, close to zero, or negative?

Please circle one of the following choices:

A. Positive B. Close to zero C. Negative

(c) Interpret your results from parts (a) and (b).