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**Problem 1.** Show that any constant Riemannian metric on $\mathbb{R}^2$ i.e. a metric of the form

$$g = a\, dx \otimes dx + 2b\, dx \odot dy + c\, dy \otimes dy,$$

where $a, b, c$ are real numbers such that $a > 0$ and $ac - b^2 > 0$, is isometric to the Euclidean metric.

**Problem 2.** Let $B = \{x^2+y^2 < 1\}$ be the unit disk. Compute the volume of $B$ with respect to the hyperbolic metric

$$g = \frac{4}{(1-x^2-y^2)^2}\left(dx \otimes dx + dy \otimes dy\right).$$

**Problem 3** Using the spherical coordinates in $\mathbb{R}^3$, identify $\mathbb{R}^3 \setminus \{0\}$ as a warped product of $\mathbb{R}_+ \times S^2$.

**Problem 4** Prove that the antipodal mapping $A : S^2 \to S^2$ given by $A(p) = -p$ is an isometry of $S^2$ with the round metric.

**Problem 5** Consider the upper half plane $\{(x, y) \in \mathbb{R}^2 | \, y > 0\}$ with the group operation :

$$(a, b)\dot{(}c, d) = (a + bc, bd).$$

The identity element is $(0, 1)$.

Prove that the left-invariant metric, which at the identity element coincides with the Euclidean metric, is given by

$$g = \frac{1}{y^2}\left(dx \otimes\ dx + dy \otimes\ dy\right).$$

Putting $z = x + \sqrt{-1}y$. Prove that the transformation

$$z \to \frac{az+b}{cz+d},$$

where $ad - bc = 1$ and $a, b, c, d \in \mathbb{R}$ is an isometry.

**Problem 6** Consider the round metric induced by the Euclidean metric on $S^2$ :

$$g = d\theta \otimes d\theta + sin^2(\theta)\, d\alpha \otimes d\alpha.$$

Compute $(\frac{\partial}{\partial\alpha})^\flat$ and $(d\theta)^\sharp$ where $\flat$ and $\sharp$ are the musical isomorphism induced by $g$.

Compute $g(d\theta, d\theta)$ and $g(d\theta \wedge d\alpha, d\theta \wedge d\alpha)$