BRONX COMMUNITY COLLEGE
of the City University of New York
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
MTH34 Review Sheet

Definitions and Statements of Theorems to be memorized:

(1) Existence and Uniqueness theorem for a first order linear ordinary differential equation (Theorem 2.4.1).
(2) Existence and Uniqueness theorem for a first order general (noninear) differential equation (Theorem 2.4.2).
(3) Existence and Uniqueness theorem for a second order linear ordinary differential equation (Theorem 3.2.1).
(4) Abel's Theorem (Theorem 3.2.7)
(5) Existence and Uniqueness theorem for an n-th order linear differential equation (Theorem 4.1.1).
(6) Definition of a fundamental set of solutions (page 223 of our textbook).
(7) Definition of a linearly dependent functions (page 223 of our textbook).
(8) Theorem on the consequence of non-zero Wronskian (Theorem 4.1.2).
(9) Theorem on the equivalence of a fundamental set of solutions of a linear ODE and its linear independence (Theorem 4.1.3).
(10) Theorem on the existence of the Laplace transform of a function (Theorem 6.1.2).
(11) Definition of the standard inner product of two real-valued functions u, v on the interval $\alpha \le x \le \beta$.
(12) Definition of the orthogonality of two real valued functions u, v on the interval $\alpha \le x \le \beta$.
(13) Definition of the Euler-Fourier formulas for the coefficients in a Fourier series.
(14) Definition of a piecewise continuous function.
(15) The Fourier Convergence Theorem (Theorem 10.3.1).
(16) Formula for the Fourier cosine series of an even function.
(17) Formula for the Fourier sine series of an odd function.

Be able to calculate and answer:

(1) Draw a direction field for the given differential equation, and based on the direction field, determine the behavior of y as $t \to \infty$:
$$y' = -y(5-y)$$
(2) Solve the following IVP and draw a few solution curves based on the initial conditions:
$$\frac{dy}{dt} = 2y - 10, \qquad y(0) = y_0$$
(3) Determine the order of the given differential equation, and state whether the equation is linear or nonlinear:
   (a) $\dfrac{dy}{dt} + ty^2 = 0.$
   (b) $\dfrac{d^3y}{dt^3} + t\dfrac{dy}{dt} + (\cos^2(t))y = t^3.$
   (c) $u_{xx} + y_{yy} + uu_x + uu_y + u = 0.$
(4) Use integrating factors to find the solution of the given IVP:
   (a) $y' + 2y = te^{-2t}, \qquad y(1) = 0.$
   (b) $y' + (2/t)y = (\cos t)/t^2, \qquad y(\pi) = 0, t > 0.$
   (c) $ty' + (t+1)y = t, \qquad y(\ln 2) = 1, t > 0.$
(5) Find the solution of the given IVP in explicit form:
   (a) $y' = (1-2x)/y, \qquad y(1) = -2.$
   (b) $dr/d\theta = r^2/\theta, \qquad r(1) = 2.$
   (c) $y' = (e^{-x} - e^x)/(3+4y), \qquad y(0) = 1.$
   (d) $y' = 2x/(y + x^2y), \qquad y(0) = -2.$
(6) **(Homogeneous equation: Use** $v = y/x$**)** Solve
   (a) $\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2xy}.$
   (b) $\dfrac{dy}{dx} = -\dfrac{4x+3y}{2x+y}.$

(7) A tank with a capacity of 500 gal originally contains 200 gal of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.

(8) A young person with no initial capital invests k dollars per year at an annual rate of return r. Assume that investments are made continuously and that the return is compounded continuously.
   (a) Determine the sum $S(t)$ accumulated at any time t.
   (b) If $r = 7.5\%$, determine k so that \$ 1 million will be available for retirement in 40 years.
   (c) If $k = \$2000$ per year, determine the return rate r that must be obtained to have\$ 1 million available in 40 years.

(9) Without solving the problem, determine an interval in which the solution of the IVP is certain to exist:
   (a) $(4 - t^2)y' + 2ty = 3t^2, \qquad y(-3) = 1.$
   (b) $(\ln t)y' + y = \cot t, \qquad y(2) = 3.$
   (c) $y'' + (\cos t)y' + 3(\ln |t|)y = 0, \qquad y(2) = 3, y'(2) = 1.$

(10) Discuss the existence and uniqueness theorem for the following:
   (a) $y' = (1 - t^2 - y^2)^{1/2}.$
   (b) $y' = \dfrac{(\cot t)y}{(1 + y)}.$

(11) **Bernoulli Equation: Use** $v = y^{1-n}$ Solve
   (a) $t^2y' + 2ty - y^3 = 0, \qquad t > 0.$
   (b) $y' = ry - ky^2, \qquad r > 0, k > 0.$

(12) Sketch the graph of $f(y)$ versus y, determine the critical (equilibrium) points, and classify each one asymptotically stable, unstable, or semistable. Draw the phase line, and sketch several graphs of solutions in the ty-plane:
   (a) $dy/dt = -k(y - 1)^2, \qquad k > 0, -\infty < y_0 < \infty.$
   (b) $dy/dt = y(1 - y^2), \qquad -\infty < y_0 < \infty.$
   (c) $dy/dt = y^2(4 - y^2), \qquad -\infty < y_0 < \infty.$

(13) Determine whether each of the equations is exact. If it is exact, find the solution:
   (a) $(e^x \sin y - 2y \sin x) + (e^x \cos y + 2 \cos x)y' = 0.$
   (b) $(y/x + 6x) + (\ln(x) - 2)\, y' = 0, \qquad x > 0.$
   (c) $(9x^2 + y - 1) - (4y - x)y' = 0, \qquad y(1) = 0.$

(14) Find the Wronskian of the given pair of functions:
   (a) e^{-2t}, te^{-2t}
   (b) $\cos^2 \theta, 1 + \cos 2\theta$

(15) Find the Wronskian of two solutions without solving the equation:
   (a) $t^2y'' - t(t + 2)y' + (t + 2)y = 0$
   (b) $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$

(16) Solve the given IVP:
   (a) $y'' + 3y' = 0, \qquad y(0) = -2, y'(0) = 3$
   (b) $4y'' - y = 0, \qquad y(-2) = 1, y'(-2) = -1$
   (c) $y'' + 4y' + 5 = 0, \qquad y(0) = 1, y'(0) = 0$
   (d) $y'' + 2y' + 2y = 0, \qquad y(\pi/4) = 2, y'(\pi/4) = -2$
   (e) $9y'' + 6y' + 82y = 0, \qquad y(0) = -1, y'(0) = 2$
   (f) $y'' + 4y' + 4y = 0, \qquad y(-1) = 2, y'(-1) = 1$
   (g) $4y^{(3)} + y' + 5y = 0, \qquad y(0) = 2, y'(0) = 1, y''(0) = -1$

(17) (Euler equations (Use $x = \ln t$)):
   (a) $t^2y'' - 3ty' + 4y = 0$ for $t > 0$.
   (b) $t^2y'' + 3ty' + y = 0$ for $t > 0$.

(18) Use the method of undetermined coefficients to find the solution of the given IVP:
  (a) $y'' + y' - 2y = 2t$, $y(0) = 0, y'(0) = 1$.
  (b) $y'' - 2y' - 3y = 3te^{2t}$, $y(0) = 1, y'(0) = 0$.
  (c) $y'' + 2y' + 5y = 4e^{-t}\cos 2t$, $y(0) = 1, y'(0) = 0$.

(19) Use the method of variation of parameters to find the general solution of
  (a) $y'' - 2y' + y = e^t/(1+t^2)$
  (b) $4y'' + y = 2\sec(t/2)$ for $-\pi < t < \pi$
  (c) $y'' + 4y' + 4y = t^{-2}e^{-2t}$ for $t > 0$

(20) A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/s and if there is no damping determine the position u of the mass at any time t. When does the mass first return to its equilibrium position?

(21) A series circuit has a capacitor of 0.25×10^{-6} F and an inductor of 1 H. If the initial charge on the capacitor is 10^{-6} C and there is no initial current, find the charge Q on the capacitor at any time t.

(22) A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10\sin(t/2)$ N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s,
  (a) Formulate the initial value problem describing the motion of the mass.
  (b) Find the solution of the initial value problem.
  (c) Identify the transient and steady state parts of the solution.
  (d) Plot the graph of the steady state solution.
  (e) If the given external force is replaced by a force of $2\cos(\omega t)$ of frequency ω, find the value of ω for which resonance occurs.

(23) Verify that the given functions are solutions of the differential equation and determine their Wronskian:
  (a) $y^{(4)} + y'' = 0$; $\quad 1, t, \cos t, \sin t$.
  (b) $x^3y''' + x^2y'' - 2xy' + 2y = 0$; $\quad x, x^2, 1/x$.

(24) Find the general solution of
  (a) $y^{(5)} - 3y^{(4)} + 3y^{(3)} - 3y^{(2)} + 2y' = 0$
  (b) $y^{(8)} + 8y^{(4)} + 16y = 0$

(25) Determine the radius of convergence of
  (a) $\sum_{n=0}^{\infty} 2^n x^n$
  (b) $\sum_{n=1}^{\infty} \dfrac{n!x^n}{n^n}$

(26) Determine the Taylor series and its radius of convergence about the point x_0 of
  (a) $\ln x$ for $x_0 = 1$
  (b) x^2 for $x_0 = -1$

(27) Seek power series solutions; find the recurrence relation; find the first four terms in each of two solutions y_1, y_2; By evaluating the Wronskian $W(y_1, y_2)(x_0)$, show that y_1, y_2 form a fundamental set of solutions; if possible, find the general term in each solution.
  (a) $y'' + xy' + 2y = 0$ for $x_0 = 0$; $y(0) = 4, y'(0) = -1$
  (b) $(4 - x^2)y'' + 2y = 0$ for $x_0 = 0$.

(28) **(Euler Equations:)** Find the solution of the given problems:
  (a) $2x^2y'' - 4xy' + 6y = 0$.
  (b) $x^2y'' - 3xy' + 4y = 0$ with $y(-1) = 2, y'(-1) = 3$.
  (c) $x^2y'' + 3xy' + 5y = 0$ with $y(1) = 1, y'(1) = -1$.

(29) Show that the given differential equation has a regular singular point at $x = 0$; determine the indicial equation, the recurrence relation, and the roots of the indicial equation; find the series solution $x > 0$ corresponding to the larger root.
  (a) $3x^2y'' + 2xy' + x^2y = 0$
  (b) $2x^2y'' + 3xy' + (2x^2 - 1)y = 0$

(30) Find the Laplace transform of

(a) $(t) = t$
(b) $f(t) = t^2$
(c) $f(t) = \cosh bt$
(d) $f(t) = \sinh bt$

(31) Use Laplace transform to find the solution of
(a) $y^{(4)} - 4y^{(3)} + 6y^{(2)} - 4y' + y = 0;\ y(0) = 0, y'(0) = 1, y^{(2)}(0) = 0, y^{(3)}(0) = 1$
(b) $y^{(2)} - 2y' + 2y = \cos t;\ y(0) = 1, y'(0) = 0$

(32) Express $f(t)$ in terms of the unit step function $u_c(t)$:

$$(a) \qquad f(t) = \begin{cases} t^2, & 0 \leq t < 2 \\ 1, & t \geq 2 \end{cases} \qquad\qquad (b) \qquad f(t) = \begin{cases} t, & 0 \leq t < 2, \\ 2, & 2 \leq t < 5, \\ 7-t & 5 \leq t < 7, \\ 0, & t \geq 7 \end{cases}$$

(33) Find the solution of the given IVP.
(a) $y'' + 3y' + 2y = u_2(t);\ y(0) = 0, y'(0) = 1$
(b) $y'' + y' + \frac{5}{4}y = g(t);\ y(0) = 0, y'(0) = 0$; where

$$g(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ 0 & t \geq \pi \end{cases}$$

(34) Solve the given boundary value problem or show that it has no solution:
(a) $y'' + 2y = x,\ y(0) = 0, y(\pi) = 0.$
(b) $y'' + 3y = \cos(x),\ y'(0) = 0, y'(\pi) = 0.$

(35) Find the eigenvalues and eigenfunctions of the given boundary value problem. Assume that all eigenvalues are real.
(a) $y'' + \lambda y = 0,\ y'(0) = 0, y'(\pi) = 0.$
(b) $y'' + \lambda y = 0,\ y'(0) = 0, y'(L) = 0.$

(36) For each of the given periodic functions, sketch the graph of each for three periods, and then find the Fourier series.
(a) $f(x) = \begin{cases} x+1, & -1 \leq x < 0, \\ 1-x, & 0 \leq x < \pi; \end{cases},\ f(x+2) = f(x).$
(b) $f(x) = \begin{cases} x+2, & -2 \leq x < 0, \\ 2-2x, & 0 \leq x < 2; \end{cases},\ f(x+4) = f(x).$

(37) Let $f(x) = 1 - x^2$ for $-1 \leq x < 1$, and extend it periodically outside the interval $[-1, 1)$. Find the Fourier series for the extended function, and sketch the graph of the function to which the series converges for three periods.

(38) Extend the function $f(x)$ periodically beyond $[-2, 2)$:

$$f(x) = \begin{cases} x+2, & -2 \leq x < 0, \\ 2-2x, & 0 \leq x < 2. \end{cases}$$

(a) Find the Fourier series for the given function.
(b) Let $e_n(x) = f(x) - s_n(x)$. Find the least upper bound or the maximum value (if it exists) of $|e_n(x)|$ for $n = 10, 20$, and 40.
(c) If possible, find the smallest n for which $|e_n(x)| \leq 0.01$ for all x.

(39) Find the Fourier cosine series for the given function:

$$f(x) = L - x. \qquad 0 \leq x < L,\ \text{period } 2L,$$

(40) Find the Fourier sine series for the given function:

$$f(x) = L - x. \qquad 0 < x < L,\ \text{period } 2L,$$

(41) Find the solution of the heat conduction problem:

$$u_{xx} = 4u_t, \qquad 0 < x < 2, t > 0;$$
$$u(0,t) = 0, \qquad u(2,t) = 0, t > 0;$$
$$u(x,0) = 2\sin(\pi x/2) - \sin(\pi x) + 4\sin(2\pi x), \qquad 0 \le x \le 2.$$

(42) Consider the conduction of heat in a rod 40 cm in length whose ends are maintained at 0^o C for all $t > 0$. Find an expression for the temperature $u(x,t)$ if the initial temperature distribution in the rod is given by the function

$$u(x,0) = \begin{cases} x, & 0 \le x < 20, \\ 40 - x, & 20 \le x \le 40. \end{cases}$$

Short answers:

(1) $y = 0$ and $y = 5$ are equilibrium solutions; y diverges from 5 if initial value is greater than 5; $y \to 0$ if initial value is less than 5.

(2) $y = 5 + (y_0 - 5)e^{2t}$. Equilibrium solution is $y = 5$; solutions diverge from the equilibrium solution.

(3) (a) First order, nonlinear, ordinary differential equation.
 (b) Third order, linear, ordinary differential equation.
 (c) Second order, nonlinear, partial differential equation.

(4) (a) $y = (t^2 - 1)e^{-2t}/2$
 (b) $y = (\sin(t))/t^2$
 (c) $y = (t - 1 + 2e^{-t})/t$, $t \neq 0$.

(5) (a) $y = -\sqrt{2x - 2x^2 + 4}$
 (b) $r = 2/(1 - 2\ln(\theta))$
 (c) $y = -\frac{3}{4} + \frac{1}{4}\sqrt{65 - 8e^t - 8e^{-t}}$.
 (d) $y = -[2\ln(1 + x^2) + 4]^{1/2}$.

(6) (a) $x^2 + y^2 - cx^3 = 0$.
 (b) $|y + x||y + 4x|^2 = c$.

(7) $Q(t) = 200 + t - [100(200)^2/(200 + t)^2]$ lb, $t < 300$; $c = 121/125\ lb/gal$; $\lim_{t\to infty} c = 1\ lb/gal$.

(8) (a) $k(e^{rt} - 1)/r$.
 (b) $k \cong \$\ 3930$.
 (c) 9.77% .

(9) (a) $(-\infty, -2)$.
 (b) $(1, \pi)$.
 (c) $(0, \infty)$.

(10) (a) Discussion should cover the region $t^2 + y^2 < 1$.
 (b) Discussion should cover the region $t \neq n\pi$ for $n \in \mathbb{Z}$ and $y \neq -1$.

(11) (a) $y = \pm[5t/(2 + 5ct^5)]^{1/2}$.
 (b) $y = r/(k + cre^{-rt})$.

(12) (a) $y = 1$ is semistable.
 (b) $y = -1$ and $y = 1$ are asymptotically stable, $y = 0$ is unstable.
 (c) $y = 2$ is asymptotically stable, $y = 0$ is semistable, $y = -2$ is unstable.

(13) (a) Exact. Solution is: $e^x \sin(y) + 2y\cos(x) + c = 0$.
 (b) Exact. Solution is: $y\ln(x) + 3x - 2y + c = 0$.
 (c) Exact. Solution of IVP is: $3x^3 + xy - x - 2y^2 - 2 = 0$.

(14) (a) e^{-4t}.
 (b) 0.

(15) (a) ct^2e^t.
 (b) $c/(1 - x^2)$.

(16) (a) $y = -1 - e^{-3t}$; $y \to -1$ as $t \to \infty$.

(b) $y = -\frac{1}{2}e^{(t+2)/2} + \frac{1}{2}e^{-(t+2)/2}$; $y \to -\infty$ as $t \to \infty$.
(c) $y = e^{-2t}\cos(t) + 2e^{-2t}\sin(t)$; decaying oscillation.
(d) $y = \sqrt{2}e^{-(t-\pi/4)}\cos(t) + \sqrt{2}e^{-(t-\pi/4)}\sin(t)$; decaying oscillation.
(e) $y = -e^{-t/3}\cos(3t) + \frac{5}{9}e^{-t/3}\sin(3t)$, $y \to 0$ as $t \to \infty$.
(f) $y = 7e^{-2(t+1)} + 5te^{(t+1)}$, $y \to 0$ as $t \to \infty$.
(g) $y = \frac{2}{13}e^{-t} + \frac{24}{13}e^{t/2}\cos(t) + \frac{3}{13}e^{t/2}\sin(t)$.

(17) (a) $y = c_1 t^2 + c_2 t^2 \ln(t)$.
(b) $y = c_1 t^{-1} + c_2 t^{-1}\ln(t)$.

(18) (a) $y = e^t - \frac{1}{2}e^{-2t} - t - \frac{1}{2}$.
(b) $y = e^{3t} + \frac{2}{3}e^{-t} - \frac{2}{3}e^{2t} - te^{2t}$.
(c) $y = e^{-t}\cos(2t) + \frac{1}{2}e^{-t}\sin(2t) + te^{-t}\sin(2t)$.

(19) (a) $y = c_1 e^t + c_2 te^t - \frac{1}{2}e^t\ln(1+t^2) + te^t\arctan(t)$.
(b) $y = c_1\cos(t/2) + c_2\sin(t/2) + t\sin(t/2) + 2[\ln(\cos(t/2))]\cos(t/2)$.
(c) $y = c_1 e^{-2t} + c_2 te^{-2t} - e^{-2t}\ln(t)$.

(20) $u = \dfrac{5}{7}\sin(14t)$ cm, t in s; $t = \pi/14\ s$.

(21) $Q = 10^{-6}\cos(2000t)\ C$, t in s.

(22) (a) $u'' + 10u' + 98u = 2\sin(t/2)$, $u(0) = 0, u'(0) = 0$, where u is in ft. and t in s.
(b) $u = \dfrac{1}{153281}[160e^{-5t}\cos(\sqrt{73}t) + \dfrac{383443}{7300}e^{-5t}\sin(\sqrt{73}t) - 160\cos(t/2) + 3128\sin(t/2)]$.
(c) The first two terms are the transient.
(d) Plot.
(e) $\omega = 4\sqrt{3}\ rad/s$.

(23) Verifications.

(24) (a) $y = c_1 + c_2 e^t + c_3 e^{2t} + c_4\cos(t) + c_5\sin(t)$.
(b) $y = e^t[(c_1 + c_2 t)\cos(t) + (c_3 + c_4 t)\sin(t)] + e^{-t}[(c_5 + c_6 t)\cos(t) + (c_7 + c_8 t)\sin(t)]$.

(25) (a) $\rho = 1/2$.
(b) $\rho = e$.

(26) (a) $\sum_{n=1}^{\infty}(-1)^{n+1}\dfrac{(x-1)^n}{n}$, $\rho = 1$.
(b) $1 - 2(x+1) + (x+1)^2$, $\rho = \infty$.

(27) (a) $a_{n+2} = -a_n/(n+1)$, $n = 0, 1, 2, \ldots$.

$$y_1(x) = 1 - x^2 + \frac{x^4}{1\cdot 3} - \frac{x^6}{1\cdot 3\cdot 5} + \cdots = 1 + \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{1\cdot 3\cdot 5\cdots(2n-1)}$$

$$y_2(x) = x - \frac{x^3}{2} + \frac{x^5}{2\cdot 4} - \frac{x^7}{2\cdot 4\cdot 6} + \cdots = x + \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n+1}}{2\cdot 4\cdot 6\cdots(2n)}$$

$$y(x) = 4 - x - 4x^2 + \frac{1}{2}x^3 + \frac{4}{3}x^4 + \cdots$$

(b) $4(n+2)a_{n+2} - (n-2)a_n = 0$, $n = 0, 1, 2, \ldots$

$$y_1(x) = 1 - \frac{x^2}{4}$$

$$y_2(x) = x - \frac{x^3}{12} - \frac{x^5}{240} - \frac{x^7}{2240} - \cdots - \frac{x^{2n+1}}{4^n(2n-1)(2n+1)} - \cdots.$$

(28) (a) $y = c_1|x|^{3/2}\cos(\frac{1}{2}\sqrt{3}\ln|x|) + c_2|x|^{3/2}\sin(\frac{1}{2}\sqrt{3}\ln|x|)$.
(b) $y = 2x^2 - yx^2\ln|x|$.
(c) $y = x^{-1}\cos(2\ln x)$.

(29) (a) $r(3r-1) = 0$, $a_n = -\dfrac{a_{n-2}}{(n+r)[3(n+r)-1]}$, $r_1 = 1/3, r_2 = 0$. $y(x) = x^{1/3}\left[1 - \dfrac{1}{1!7}\left(\dfrac{x^2}{2}\right) + \dfrac{1}{2!7\cdot 13}\left(\dfrac{x^2}{2}\right)^2 + \cdots +\right.$

(b) $2r^2 + r - 1 = 0$, $(2n + 2r - 1)(n + r + 1)a_n + 2a_{n-2} = 0$, $r_1 = 1/2, r_2 = -1$.

$$y(x) = x^{1/2}\left(1 - \frac{x^2}{7} + \frac{x^4}{2!7\cdot 11} - \cdots + \frac{(-1)^m x^{2m}}{m!7\cdot 11\cdots(4m+3)} + \cdots\right).$$

(30) (a) $F(s) = 1/s^2, s > 0$.
(b) $F(s) = 2/s^3, s > 0$.
(c) $F(s) = \dfrac{s}{s^2 - b^2}, s > |b|$.

(d) $F(s) = \dfrac{b}{s^2 - b^2}, s > |b|$.

(31) (a) $y = te^t - t^2e^t + \dfrac{2}{3}t^3e^t$.

(b) $y = \dfrac{1}{5}(\cos(t) - 2\sin(t) + 4e^t\cos(t) - 2e^t\sin(t))$.

(32) (a) $f(t) = t^2 + u_2(t)(1 - t^2)$.

(b) $f(t) = t + u_2(t)(2 - t) + u_5(t)(5 - t) - u_7(t)(7 - t)$.

(33) (a) $y = e^{-t} - e^{-2t} + u_2(t)[\dfrac{1}{2} - e^{-(t-2)} + \dfrac{1}{2}e^{-2(t-2)}]$.

(b) $y = h(t) + y_\pi(t)h(t - \pi)$, where $h(t) = \dfrac{4}{17}[-4\cos(t) + \sin(t) + 4e^{-t/2}\cos(t) + e^{-t/2}\sin(t)]$.

(34) (a) $y = (-\pi \sin .(\sqrt{2}x) + x\sin(\sqrt{2}\pi))/2\sin(\sqrt{2}\pi)$.

(b) $y = \dfrac{1}{2}\cos(x)$.

(35) (a) $\lambda_0 = 0, y_0(x) = 1$; $\lambda_n = n^2$, $y_n(x) = \cos(nx)$; $n = 1, 2, 3, \ldots$.

(b) $\lambda_0 = 0, y_0(x) = 1$; $\lambda_n = (n\pi/L)^2$, $y_n(x) = \cos(n\pi x/L)$; $n = 1, 2, 3, \ldots$.

(36) (a) $f(x) = \dfrac{1}{2} + \dfrac{4}{\pi^2}\sum_{n=1}^\infty \dfrac{\cos(2n-1)\pi x}{(2n-1)^2}$.

(b) $f(x) = \dfrac{1}{2} + \dfrac{12}{\pi^2}\sum_{n=1}^\infty \dfrac{\cos[(2n-1)\pi x/2]}{(2n-1)^2} + \dfrac{2}{\pi}\sum_{n=1}^\infty \dfrac{(-1)^n}{n}\sin\dfrac{n\pi x}{2}$.

(37) $f(x) = \dfrac{2}{3} + \dfrac{4}{\pi^2}\sum_{n=1}^\infty \dfrac{(-1)^{n+1}}{n^2}\cos(n\pi x)$.

(38) (a) $f(x) = \dfrac{1}{2} + \sum_{n=1}^\infty \left[\dfrac{6(1 - \cos(n\pi))}{n^2\pi^2}\cos\left(\dfrac{n\pi x}{2}\right) + \dfrac{2\cos(n\pi)}{n\pi}\sin\left(\dfrac{n\pi x}{2}\right)\right]$.

(b) $n = 10$; $lub|e| = 1.0606$ as $x \to 2$.

$n = 20$; $lub|e| = 1.0304$ as $x \to 2$.

$n = 40$; $lub|e| = 1.0152$ as $x \to 2$.

(c) Not possible.

(39) $f(x) = \dfrac{L}{2} + \dfrac{4L}{\pi^2}\sum_{n=1}^\infty \dfrac{\cos[(2n-1)\pi x/L]}{(2n-1)^2}$.

(40) $f(x) = \dfrac{2L}{\pi}\sum_{n=1}^\infty \dfrac{\sin(n\pi x/L)}{n}$.

(41) $u(x,t) = 2e^{-\pi^2 t/16}\sin(\pi x/2) - e^{-\pi^2 t/4}\sin(\pi x) + 4e^{-\pi^2 t}\sin(2\pi x)$.

(42) $u(x,t) = \dfrac{160}{\pi^2}\sum_{n=1}^\infty \dfrac{\sin(n\pi/2)}{n^2}e^{-\pi^2 n^2 t/1600}\sin\left(\dfrac{n\pi x}{40}\right)$.