Bronx Community College of CUNY
Department of Mathematics and Computer Science
MTH 32 Review

1. Find the area of the region bounded by the curves $y = x^2 - 2x$ and $y = x$.

2. Find the area between the curves $y = x^2 + 4$, and $y = 12 - x^2$.

3. Find the area of the region bounded by the curves $y = x^3$, $y = x$.

4. Find the area of the region bounded by the curves $x = 1 - y^2$, and $x = y^2 - 1$.

5. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$, $y = \sqrt{x}$, about a) The x-axis b) the y-axis c) about $y = 2$.

6. Find the volume of the solid obtained by rotating the region bounded above by the curve $y = 2x - x^2$ and below by the x-axis about the y-axis.

7. Find the volume of the solid obtained by rotating the region bounded by $y = e^{-x}$, $y = 0$ and $x = 0$ about a) the x-axis b) about the y-axis.

8. Find the volume of the solid obtained by rotating the region bounded by $y = \ln x$, $y = 1$, $y = 5$, $x = 0$ about the y-axis.

9. Find the volume of the solid obtained by rotating the curve $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 9$, about the y-axis.

10. Find the volume of the solid obtained by rotating the region bounded by the curves $xy = 1$, $x = 0$, $y = 1$, $y = 3$, about the x-axis.

11. Compute the integrals:

a) $\displaystyle\int_0^{1/2} \frac{1}{1+2x}dx$

b) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

c) $\displaystyle\int \tan x \ln(\cos x)dx$

d) $\displaystyle\int_1^2 \frac{\ln x}{x}dx$

e) $\displaystyle\int_{-\pi/3}^{\pi/3} x^4 \sin x dx$

f) $\displaystyle\int_{-1}^{1} \frac{\tan x}{1+x^2}dx$

g) $\displaystyle\int_1^2 \frac{(x+1)^2}{x}dx$

h) $\displaystyle\int \frac{x+1}{x^2+1}dx$

i) $\int (\cos x + \sin x)^2 dx$

j) $\int_1^2 \frac{x}{(x+1)^2} dx$

k) $\int_0^{\pi/2} \sin^3 x \cos^2 x dx$

l) $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x} dx$

m) $\int_0^{\pi/3} \cos^2(3x) dx$

n) $\int \sin^{-1}(x) dx$

o) $\int x \sin(5x) dx$

p) $\int \sin^4 x dx$

q) $\int \cos^5 x dx$

r) $\int \tan^5 x \sec^3 x dx$

s) $\int w \ln w dw$

t) $\int (\ln x)^2 dx$

u) $\int e^{3x}\ cosxdx$

v) $\int \frac{dx}{x^2\sqrt{16-x^2}}$

w) $\int x \sin(4x)\cos(4x) dx$

x) $\int \frac{1}{\sqrt{9-4x^2}} dx$

y) $\int \frac{dx}{x\sqrt{x^2+1}}$

z) $\int \frac{dx}{x^2\sqrt{x^2-9}}$

A) $\int \frac{x-1}{x^2+2x} dx$

B) $\int \frac{1}{x^2-4x+3} dx$

C) $\int \frac{1}{x^3+x} dx$

D) $\int \frac{1}{x^2-4x+5} dx$

E) $\int e^{\sqrt{x}} dx$

12. Determine whether the following improper integrals converge or diverge.

a) $\int_1^\infty \frac{1}{\sqrt{x}}dx$

b) $\int_0^\infty e^{-x}dx$

c) $\int_0^1 \frac{1}{\sqrt{x}}dx$

d) $\int_2^\infty \frac{1}{x\ln x}dx$

e) $\int_1^\infty \frac{\ln x}{x^2}dx$

f) $\int_0^1 \frac{x-1}{\sqrt{x}}dx$

g) $\int_0^4 \frac{\ln x}{\sqrt{x}}dx$

13. Find the length of the following curves:

a) $y = \frac{2}{3}x^{3/2}, \ 0 \le x \le 2$

b) $y = \frac{x^3}{3} + \frac{1}{4x}, \ 1 \le x \le 2$

c) $y = \frac{2}{3}(1+x^2)^{3/2}, \ 0 \le x \le 1$

d) $y = 2\ln(\sin(\frac{x}{2})), \ \frac{\pi}{2} \le x \le \pi$

e) $x = \frac{y^4}{8} + \frac{1}{4y^2}, \ 1 \le y \le 2$

14. Compute the limit of the following sequences as $n \to \infty$ to determine whether they converge or diverge.

a) $a_n = \frac{\cos n}{n^2}$

b) $a_n = \frac{n \sin n}{\sqrt{n^3 - 1}}$

c) $a_n = \frac{\tan^{-1} n}{e^{\sqrt[n]{n}}}$

d) $a_n = \ln(n+1) - \ln(n)$

e) $a_n = \left(\frac{n}{n+2}\right)^n$

15. The following series converge. Compute their sum.

a)

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+3}\right)$$

b)

$$\sum_{n=1}^{\infty} (\tan^{-1}(n+1) - \tan^{-1}(n))$$

c)

$$\sum_{n=0}^{\infty} \frac{2^n}{5^{n+1}}$$

d)

$$\sum_{n=0}^{\infty} (-1)^n 6^{-2n-2}$$

e)

$$\sum_{n=0}^{\infty} \frac{1-2^n}{3^n}$$

16. Suppose $\sum_{n=1}^{\infty} a_n$ is a series where $a_n > 0$ for $n \geq 1$, and $\sum_{n=1}^{\infty} a_n = 0.9$. Let $s_n = a_1 + a_2 + \ldots + a_n$. Which of the following statements are true? Which are false? Explain your reasoning.

(a) $\lim_{n\to\infty} a_n = 0.9$
(b) $\lim_{n\to\infty} s_n = 0.9$
(c) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges
(d) If $b_n = a_n + 0.1$ then $\sum_{n=1}^{\infty} b_n$ converges.
(e) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.
(f) $\sum_{n=1}^{\infty} (\sin n) a_n$ converges.

17 . Determine whether the following series converge or diverge. Clearly state the test you are using and the evidence backing your conclusion.

a)

$$\sum_{n=0}^{\infty} \frac{5}{2^n + 1}$$

b)

$$\sum_{n=1}^{\infty} \frac{n^4}{4^n}$$

c)

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n^2}\right)$$

d)

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^3 + \sqrt{n}}$$

e)

$$\sum_{n=2}^{\infty} \frac{\ln n}{\ln(n^5)}$$

f)

$$\sum_{n=2}^{\infty} \frac{n}{\ln n}$$

g)

$$\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^{n-1}$$

h)

$$\sum_{n=1}^{\infty} \frac{2n^2+1}{3n^2+1}$$

i)

$$\sum_{n=0}^{\infty} \frac{3^n}{4^n n!}$$

j)

$$\sum_{n=5}^{\infty} \frac{\sqrt{5}}{n(n-4)}$$

k)

$$\sum_{n=0}^{\infty} \frac{\cos n}{(1.1)^n}$$

l)

$$\sum_{n=2}^{\infty} \frac{2\sqrt[3]{n}}{n^2-1}$$

m)

$$\sum_{n=0}^{\infty} \left(\frac{2n^2+1}{3n^2+1}\right)^n$$

n)

$$\sum_{n=0}^{\infty} ne^{-n^2}$$

o)

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$$

p)

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$$

q)

$$\sum_{n=1}^{\infty} (\sqrt[n]{2}-1)^n$$

18. Determine whether the following series converge absolutely, converge conditionally or diverge.

a)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n^2+1}}$$

b)

$$\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{2n}$$

c)

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^2}$$

d)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{5n}}$$

19. Find the center, radius and interval of convergence of the following power series. If the interval of convergence has endpoints, please determine whether or not the series converges at the endpoints of the interval.

a)

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n4^n}$$

b)

$$\sum_{n=1}^{\infty} \frac{x^n}{n\ln(n+1)}$$

c)

$$\sum_{n=1}^{\infty} \frac{(2x-5)^n}{10^{n+1}}$$

20. Let $f(x) = \sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$

a) Find the center, radius and interval of convergence of the above power series. If the interval of convergence has endpoints, please determine whether or not the series converges at the endpoints of the interval.

b) Compute the power series for $f'(x)$, the derivative of f. Find the radius and interval of convergence of this series.

c) Compute the power series for $g(x) = \int f(x)dx$. (Assume that $g(2) = 0$, which implies that the constant of integration is 0.) Find the radius and interval of convergence of this series.

d) Answer questions a, b,c for $g(x) = \sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n}}$

21. a) What is the radius of convergence of the series $\sum_{n=1}^{\infty} n2^n x^{n-1}$?

b) What is the Maclaurin series for $\frac{1}{1-2x}$?

What is the radius and interval of convergence for this series?

c) Use your answer to part b) to determine the sum of the series in part a) as a function of x, whenever it converges.

22. a) Find a power series representation for $\ln(1+x)$. What is the radius and interval of convergence of this series?

b) Use your answer to a) to compute the sum

$$\frac{2}{3} - \frac{2^2}{2.3^2} + \frac{2^3}{3.3^3} - \frac{2^4}{4.3^4} + \ldots$$

c) Use your answer to a) to also compute the sum

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^n}{n5^n}$$

23. Find the Maclaurin series of $\int \frac{e^x}{x}dx$.

24. a) Find the Maclaurin series generated by $x\sin(x^3)$.
b) What is the radius of convergence of the series in a)?
c) Find the Maclaurin series for $\int x\sin(x^3)dx$. What is the radius of convergence of the series in c)
d) Give the first four non-zero terms of the series for $\int_0^{0.1} x\sin(x^3)dx$ (You may leave your answer as a sum).

25. a) What is the Taylor series of $\frac{1}{x^2}$ around $x = -1$?

b) For which x does this Taylor series converge? If the interval of convergence has endpoints, test them for convergence.

26. Find the second degree Taylor polynomial for
a) $f(x) = \sqrt{1+x}$ about $x = 0$.
b) $g(x) = \sqrt{x}$ about $x = 4$.
c) $h(x) = \sqrt[3]{x}$, about $x = 8$.
d) $k(x) = \cos x$, about $x = \frac{\pi}{4}$.

27. How many non-zero terms of the Maclaurin series for e^x are needed to approximate $\frac{1}{e}$ with error less than 10^{-3}?

28. The equation

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

comes from evaluating at $x = 1$ the Maclaurin series for $tan^{-1}(x)$. How many terms are needed in order to approximate $\pi/4$ with error less than 0.01?