Bronx Community College of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

Math 32 Review Sheet

1. Find the area between the curves $y = x^3$ and $y = x^4$.
2. Find the area between the curve $y = |\ln x|$, the x–axis, and the lines $y = e$ and $y = 1/e$.
3. The base of a solid is a circle with radius 1. Each cross-section perpendicular to a given diameter is a square. Find the volume of the solid.
4. Find the volume of the solid of revolution obtained by revolving the plane region bounded by $y = x^2$, the x–axis, and the line $x = 4$ about the x–axis.
5. The region bounded by $y = \dfrac{1}{x}$, $y = 0$, $x = 1$, and $x = 4$ is revolved about the y–axis. Find the volume of the resulting solid.
6. The region bounded by $y = e^{-x}$, $y = 0$ and $x = 0$ is revolved about the x–axis. Find the volume of the resulting solid.
7. The region bounded by $y = e^{-x}$, $y = 0$ and $x = 0$ is revolved about the y–axis. Find the volume of the resulting solid.
8. Differentiate and simplify your answer:
   (a) $x \ln(x^2)$
   (b) $x (\ln x)^3$
   (c) $\cos(\ln x)$
   (d) $(\sin x)(\ln x)^2$
   (e) $(\cos x) \ln(x^2)$
   (f) $\dfrac{1}{10} \ln\left(\dfrac{5+x}{5-x}\right)$
   (g) $\log_5 \sqrt[3]{x}$
   (h) $\log_2 \left[(x^2+1)^3 \sin(\beta x)\right]$
   (i) $\dfrac{x^2}{2 \log_3 x}$
   (j) $(\sin 2x)(\sin e^{-x})$
   (k) $3^{\sqrt{x}}$
   (l) $\ln e^{\cos 3\theta}$
   (m) $\dfrac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$
   (n) $\dfrac{1}{ac} \ln(b + ce^{ax})$
   (o) $\dfrac{1}{\arcsin 2x}$

(p) $x \arctan 5x - \dfrac{1}{10} \ln\left(1 + 25x^2\right)$

(q) $e^{3x} \sinh x$

(r) $\dfrac{(\tanh 3x)(\cosh 2x)}{\sqrt{1+2x}}$

9. Find the arc length of $y = x^{3/2}$ between the points $(1, 1)$ and $(4, 8)$.

10. Find the length of one arc of the cycloid $\{x = a(t - \sin t), \quad y = a(1 - \cos t)\}, \quad a > 0, \quad 0 \leq t \leq 2\pi$.

11. Integrate:

(a) $\displaystyle\int \frac{x}{1+x^2}\, dx$

(b) $\displaystyle\int \frac{1}{1+x^2}\, dx$

(c) $\displaystyle\int \frac{x^2}{1+x^2}\, dx$

(d) $\displaystyle\int \frac{x^3 + x^2 - 1}{x^2 + 1}$

(e) $\displaystyle\int \frac{\ln x}{x}\, dx$

(f) $\displaystyle\int \tan x\, dx$

(g) $\displaystyle\int e^{4x}\, dx$

(h) $\displaystyle\int x^2 e^{x^3+4}\, dx$

(i) $\displaystyle\int \frac{1}{x\sqrt{x^2-1}}\, dx$

(j) $\displaystyle\int \frac{x}{\sqrt{x^4-1}}\, dx$

(k) $\displaystyle\int \sin^4 x\, dx$

(l) $\displaystyle\int \cos^5 x\, dx$

(m) $\displaystyle\int \frac{1-x^2}{x^2}\, dx$

(n) $\displaystyle\int x^2 \sin x\, dx$

(o) $\displaystyle\int \frac{6x^2 - 15x + 22}{(x+3)(x^2+2)^2}\, dx$

(p) $\displaystyle\int \frac{1}{\sqrt{3-4x^2}}\, dx$

(q) $\displaystyle\int \frac{5}{x^2}\,dx$

(r) $\displaystyle\int \frac{1}{(x+1)(x^2+1)}\,dx$

(s) $\displaystyle\int \frac{1}{x^4-1}\,dx$

12. Evaluate the following improper integrals:

(a) $\displaystyle\int_1^\infty xe^{-x^2}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx$

(c) $\displaystyle\int_{-\infty}^\infty x^3\,dx$

(d) $\displaystyle\int_0^2 \frac{dx}{x-1}$

13. Evaluate the following limits:

(a) $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{4x}$

(b) $\displaystyle\lim_{x\to 0} \frac{\sin x - x}{x^3}$

(c) $\displaystyle\lim_{x\to 1} \frac{\ln x}{x^2-1}$

(d) $\displaystyle\lim_{x\to 0} \left(\csc x - \frac{1}{x}\right)$

(e) $\displaystyle\lim_{x\to\infty} (\ln(x+1) - \ln(x-1))$

(f) $\displaystyle\lim_{x\to 0^+} x^x$

14. Find the area enclosed by

A. $r = 4\sin 2\theta$ between 0 and 2π. B. $r = \dfrac{1}{1+\theta}$ between $\theta = \dfrac{\pi}{4}$ and $\theta = \dfrac{\pi}{2}$

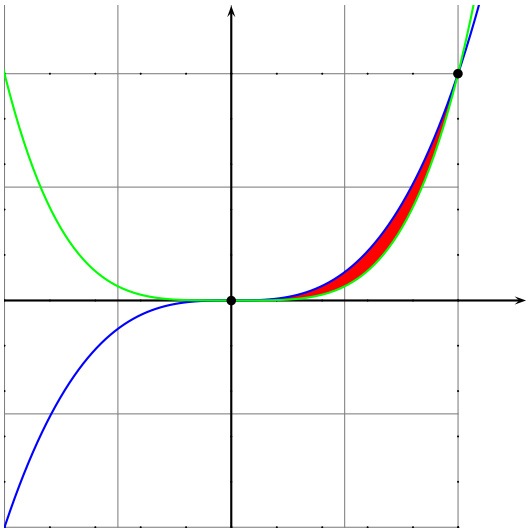
15. Find the center, foci, and vertices of $x^2 - 2x + 4y^2 + 10y = 7$.

16. Find the center, foci, vertices, and asymptotes of $x^2 = y^2 - 2y$.

17. Find the focus, directrix, symmetry axis, and vertex of $y = x^2 - 4x + 5$.

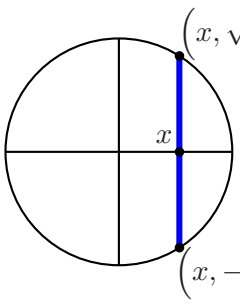
18. Find the area of the surface obtained by rotating about the x–axis that part of the curve $y = e^x$ that lies above $[0, 1]$.

The answers

1. 

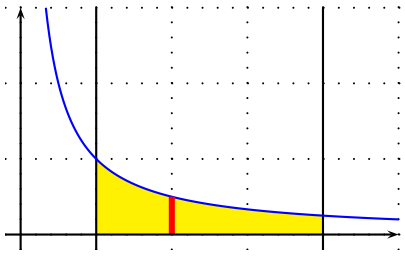
$$\int_0^1 \left|x^3 - x^4\right| \, dx = \frac{1}{20}$$

2. $\displaystyle\int_{e^{-1}}^{e} |\ln x| = 2(1 - e^{-1})$

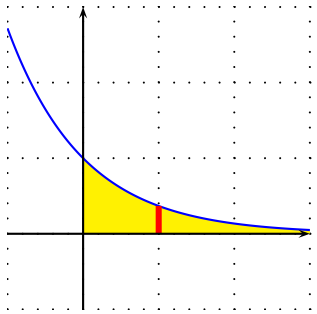
3. 

$$\int_{-1}^{1} \left(2\sqrt{1-x^2}\right)^2 \, dx = \frac{16}{3}$$

4. $\displaystyle\int_0^4 \pi \left(x^2\right)^2 \, dx = \frac{1024\pi}{5}$

5. 

$$\int_1^4 2\pi x \frac{1}{x} \, dx = 6\pi$$

6. 

$$\int_0^\infty \pi \left(e^{-x}\right)^2 \, dx = \frac{\pi}{2}$$

7. $\displaystyle\int_0^\infty 2\pi x e^{-x} \, dx = \int_0^1 \pi \left(\ln x\right)^2 \, dx = 2\pi$

8. (a) $2 + \ln\left(x^2\right)$

   (b) $(\ln x)^3 + 3(\ln x)^2$

   (c) $\displaystyle -\frac{\sin\left(\ln x\right)}{x}$

   (d) $\displaystyle (\ln x)^2 \cos x + \frac{2 \sin x \ln x}{x}$

   (e) $\displaystyle \frac{2\cos x}{x} - \ln(x^2) \sin x$

   (f) $\displaystyle \frac{1}{25 - x^2}$

   (g) $\displaystyle \frac{1}{3x \ln 5}$

   (h) $\displaystyle \frac{\beta(1+x^2)\cos\beta x + 6x \sin \beta x}{(1+x^2)\ln 2 \sin \beta x}$, or $\displaystyle \frac{1}{\ln 2}\left(\beta \tan \beta x + \frac{6x}{x^2+1}\right)$

   (i) $\displaystyle \frac{x}{\log_3 x} - \frac{x}{2 \ln 3 \left(\log_3 x\right)^2}$

   (j) $2 \cos 2x \sin e^{-x} - e^{-x} \cos e^{-x} \sin 2x$

   (k) $\displaystyle \frac{3^{\sqrt{x}} \ln 3}{2\sqrt{x}}$

   (l) $-3 \sin \theta$

   (m) $e^{ax} \sin bx$

   (n) $\displaystyle \frac{e^{ax}}{b + ce^{ax}}$

   (o) $\displaystyle \frac{-1}{2\sqrt{1-x^2}\arcsin^2 x}$

   (p) $\arctan 5x$

   (q) $e^{3x}\left(3 \sinh x + \cosh x\right)$

   (r) $\displaystyle \frac{2\cosh 2x}{\sqrt{1+2x}} - \frac{\sinh 2x}{(2x+1)\sqrt{1+2x}}$

9. $\displaystyle\int_0^4 \sqrt{1 + \left(\frac{dx^{3/2}}{dx}\right)^2} \, dx = \frac{80\sqrt{10} - 13\sqrt{13}}{27}$

10. $\displaystyle\int_0^{2\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 8a$

11. (a) $\dfrac{1}{2}\ln(x^2+1) + C$

(b) $\arctan x + C$

(c) $x - \arctan x + C$

(d) $\dfrac{x^2}{2} + x - 2\arctan x - \dfrac{\ln(x^2+1)}{2} + C$

(e) $\dfrac{(\ln|x|)^2}{2} + C$

(f) $\ln(\sec x) + C$

(g) $\dfrac{e^{4x}}{4} + C$

(h) $\dfrac{e^{x^3+4}}{3} + C$

(i) $\arctan\left(\sqrt{x^2-1}\right) + C$

(j) $\dfrac{1}{2}\ln\left(x^2 + \sqrt{x^4-1}\right) + C$

(k) $\dfrac{3}{8}x - \dfrac{1}{4}\sin 2x + \dfrac{1}{32}\sin 4x + C$

(l) $\sin x - \dfrac{2}{3}\sin^3 x + \dfrac{\sin^5 x}{5} + C$

(m) $-\dfrac{1}{x} - x + C$

(n) $2x\sin x + 2\cos x - x^2\cos x + C$

(o) $\dfrac{3}{2}\sqrt{2}\arctan\left(\dfrac{x\sqrt{2}}{2}\right) + \dfrac{1}{2}\ln\left(\dfrac{(x+3)^2}{x^2+2}\right) + \dfrac{5}{2x^2+4} + C$

(p) $\dfrac{1}{2}\arcsin\left(\dfrac{2x}{\sqrt{3}}\right) + C$

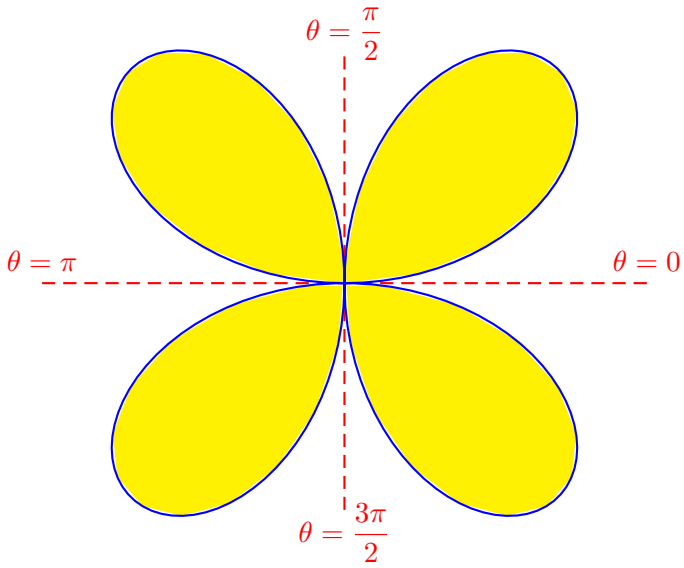
(q) $-\dfrac{5}{x} + C$

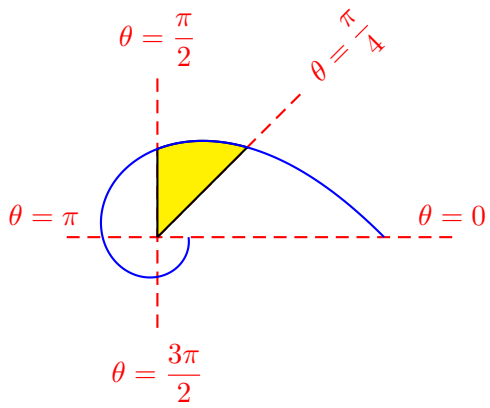
(r) $\dfrac{1}{4}\ln\left(\dfrac{(x+1)^2}{x^2+1}\right) + \dfrac{1}{2}\arctan x + C$

(s) $\dfrac{1}{4}\ln\left|\dfrac{x-1}{x+1}\right| - \dfrac{1}{2}\arctan x + C$

12. A. $\dfrac{1}{2e}$ B. π C. Divergent D. Divergent

13. A. $\dfrac{5}{4}$ B. $-\dfrac{1}{6}$ C. $\dfrac{1}{2}$ D. 0 E. 0 F. 1





14. A. 8π B. $\dfrac{\pi}{(\pi+2)(\pi+4)}$

15. The center is $(1, -\frac{5}{4})$, foci at $(1 \pm \frac{3\sqrt{19}}{4}, -\frac{5}{4})$ and vertices at $(1 \pm \frac{8\sqrt{57}}{57}, -\frac{5}{4})$.

16. The center is $(0, 1)$, foci at $(0, 1 \pm \sqrt{2})$, vertices $(0, 0)$ and $(0, 2)$, assymptotes $y = \pm x + 1$.

17. The focus is $(2, \frac{5}{4})$, directrix is $y = \frac{3}{4}$, axis $x = 2$ and vertex $(2, 1)$.

18. $\displaystyle\int_0^1 2\pi e^x \sqrt{1 + (e^x)^2}\, dx = \pi \left(\ln \frac{e + \sqrt{1+e^2}}{1+\sqrt{2}} + e\sqrt{1+e^2} - \sqrt{2} \right)$

Revised November 2007 (NEA, AKH)