Some Useful Formulas – Applications of Definite Integrals

1) The arc length L of the smooth curve given parametrically as $\begin{cases} x = g(t) \\ y = h(t) \end{cases}$, $a \le t \le b$, is given by the definite integral $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

2) If the curve is given in Cartesian coordinates by the equation $y = y(x),\ a \le x \le b,$ then the arc length is $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

3) If the curve is given in polar coordinates by the equation $r = r(\theta),\ \alpha \le \theta \le \beta,$ then the arc length is $L = \int_\alpha^\beta \sqrt{(r(\theta))^2 + (r'(\theta))^2}\,d\theta$

4) The area of a plane region between the curve $r = r(\theta),\ \alpha \le \theta \le \beta,$ in polar coordinates, and the lines $\theta = \alpha$ and $\theta = \beta$ is given by the definite integral $A = \frac{1}{2}\int_\alpha^\beta r^2(\theta)\,d\theta$

5) The surface area of the solid of revolution made up by rotating a smooth curve $y = y(x),\ a \le x \le b,$ about the *x*-axis is given by the definite integral $A = 2\pi \int_a^b y(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

6) If the curve is given parametrically as $\begin{cases} x = g(t) \\ y = h(t) \end{cases}$, $a \le t \le b,$ then the surface area of the solid of revolution is $A = 2\pi \int_a^b y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

7) If the curve is given in polar coordinates by the equation $r = r(\theta),\ \alpha \le \theta \le \beta,$ then the surface area of the solid of revolution is $A = 2\pi \int_\alpha^\beta r(\theta)\sin\theta \sqrt{(r(\theta))^2 + (r'(\theta))^2}\,d\theta$

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