

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

Review Sheet for MTH 21, Survey of Mathematics I.

This course is a survey of several topics in mathematics and its applications. The topics include: the development of numerical systems and basic laws of arithmetic in base systems; counting principles and their use in calculations of probabilities of random events; examples of optimization problems using mathematical models involving quadratic functions and solving linear programming problems involving two unknowns using the corner points method; basic formulas of financial mathematics and their applications in banking, mortgages and loans. The instructor can add other supplementary topics and can decide to treat some of the topics in greater detail. The list of problems below covers the core material of the course. Final exams are prepared individually by instructors.

MTH 21 is sometimes offered as a writing intensive course and some problems below reflect that. Problems marked with \oplus require a calculator.

Numerical Systems

1. Write 1492 and 2308 as Roman numerals.
2. Write 314 and $7/12$ in Egyptian hieroglyphs.
3. Convert 344.4_5 and ABC_{16} to base 10 decimal numbers.
4. Convert 156 and 269 to base 12 numerals.
5. Compute: $1234_5 + 4321_5$
6. Compute: $101_2 + 1011_2$
7. Write 456.17 in expanded form (as powers of 10).

Counting and Probability

8. Explain the terms: set, subset, element, cardinal number, countably infinite.
9. Evaluate
 - (a) the factorial: $5!$
 - (b) the combination: $C_{9,4}$
 - (c) the permutation: $P_{3,3}$
10. Eleven runners enter a race. Assuming they all finish at different times, how many ways can gold, silver and bronze medals be awarded?
11. How many ways can you choose a group of 4 people from 7 people?

12. \oplus All possible permutations of the first 11 letters of the alphabet are printed and all printed pages are collected in binders. If 100 permutations can be printed on one page and each binder contains 200 pages, how many binders are needed?
13. You are in a class with 19 other students. The instructor will collect homework of three randomly selected students. What is the probability that your homework will be collected?
14. You are dealt a random card from a pack of 52.
- What is the probability that this card is a club?
 - What is the probability that this card is a 4 of diamonds?
 - What is the probability that you get either a king or a queen?
 - Of parts (a), (b) and (c), which is most likely?
15. \oplus What is the probability of being dealt exactly three Jacks in a hand of 5 cards? Write the answer as a percentage.
16. The house odds that the Yankees will win the World Series this year are given as 13 : 12. If you bet \$25 that the Yankees win, and they do, how much does the house pay you? (Round to the nearest dollar.)
17. Four numbers between 1 and 10 are designated “winning numbers”. You guess a number between 1 and 10. What is the probability that you guess a winning number?
18. What is the expected value of the following game? A contestant pays \$1.50 to roll a pair of dice once. If the sum of the dice is 11, the contestant is paid \$24. Is it a fair game?
19. Explain in words the difference between the relative frequency and the theoretical probability of a random event. Use complete sentences.

Modeling with Algebra

20. The height of grain in a storage container increases by 2 feet each day. The bottom of the container is 5 feet below ground level. Write a linear equation, using the variables H and D , which gives the height H of grain above ground level after D days.
21. The feasible region for an optimization problem has corner points at $(1, 9)$, $(2, 4)$, $(11, 0)$ and $(7, 14)$. Minimize the function $P = 6x + 5y$ on this region.
22. A ball thrown off a tower follows a path given by the quadratic equation

$$h = 256 - 16t^2$$

where h is the height of the ball (measured in feet) and t is the time (measured in seconds) that the ball is in the air. After how many seconds does the ball hit the ground?

23. A cannon ball is shot into the air and follows a parabolic path given by the quadratic equation

$$y = -3x^2 + 4x.$$

Here y is the height of the cannon ball and x is the distance along the ground. Distances are measured in miles.

- (a) How many miles away from the firing point does the cannon ball land?
- (b) What is the maximum height reached by the cannon ball?

24. A corner point P of a feasible region is the intersection point of the following lines:

$$\begin{aligned} 3x - 4y &= 14 \\ x + 2y &= 3. \end{aligned}$$

What are the coordinates of P ?

25. Explain in words how to tell which side of a line to shade when graphing a linear inequality. Use complete sentences.

Financial mathematics

26. \oplus A jewelry store marks up the cost of a necklace by 35%. The price is advertised as \$315. What was the original cost of the necklace? (Round to the nearest cent.)
27. \oplus A principal of \$5,000 is invested in a bank account earning 7% compounded monthly. To the nearest cent, what is the amount in the account
- (a) after a year?
 - (b) after 4 years?
 - (c) after 100 years?
28. \oplus The daily periodic interest rate on your credit card statement is listed as 0.06573%.
- (a) If you keep the outstanding balance of \$1000 for 30 days, how much will the finance charge be?
 - (b) What will be the annual finance charge if you keep the outstanding balance of \$1000 for the whole year (365 days)?
29. \oplus If you invest a certain amount at the interest rate of $5\frac{1}{4}\%$ compounded monthly, how long do you have to wait for your investment to double?
30. \oplus Each week you deposit \$5 into an account earning 2% interest compounded weekly.
- (a) What is the amount in the account after 20 years?
 - (b) If you received no interest, what is the amount in the account after 20 years?

Answers

1. 1492 is MCDXCII and 2308 is MMCCCVIII.
2. Use three spirals, one heel bone and four staffs to represent 314. Since $7/12 = 1/3 + 1/4$ use the mouth hieroglyph over three staffs and over four staffs.
3. $344.4_5 = 99.8$, $ABC_{16} = 2748$.
4. $156 = 11_{12}$, $269 = 1A5_{12}$.

5. 11110_5
6. 110111_2
7. $456.17 = 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 1 \times 10^{-1} + 7 \times 10^{-2}$
8. See the course text.
9. (a) 120, (b) 126, (c) 6
10. $P_{11,3} = 990$ ways.
11. $C_{7,4} = 35$ ways.
12. 1996 binders.
13. $0.15 = 15\%$
14. (a) $1/4$, (b) $1/52$, (c) $8/52 = 2/13$, (d) part (a) is most likely.
15. The probability is (rounded to the nearest hundredth of a percent)
- $$\frac{C_{4,3} \times C_{48,2}}{C_{52,5}} = \frac{4512}{2598960} = 0.001736 \dots = 0.17\%$$
16. \$27
17. The probability is 0.4.
18. The expected value is -16.7 cents. Since the expected value is not zero, the game is not fair.
19. See the course text.
20. $H = 2D - 5$
21. The minimum value 32 is attained at $(2, 4)$.
22. The ball hits the ground after 4 seconds.
23. (a) it lands $1\frac{1}{3}$ miles away, (b) maximum height is $1\frac{1}{3}$ miles.
24. $P = (4, -1/2)$
25. See the course text.
26. The original cost was \$233.33
27. (a) \$5,361.45, (b) \$6,610.27, (c) \$5,372,777.60 (yes, that's correct!)
28. (a) \$19.90, (b) \$271.04
29. You have to wait 13 years and 3 months for your money to double.
30. (a) \$6392.23, (b) \$5200.00

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