1. Perform the indicated operations and simplify:

(a) \( \frac{n^2 - 3n - 10}{n^2 + n - 2} \)

(b) \( \frac{1 + \frac{5}{3y}}{\frac{6}{2x} + \frac{5}{y}} \)

(c) \( \frac{2x^2 - x}{2x^2 + x - 1} \)

(d) \( \left( \frac{x^2 - 2xy - 3y^2}{x^2 - 9y^2} \right) \left( \frac{2x^2 - xy}{2x^2 + xy - y^2} \right) \)

(e) \( \frac{9ab}{3a + b} \div \frac{a^2 - 6a}{a^2 - 4a - 12} \)

(f) \( \frac{10}{x^2 - 5x} + \frac{2}{x} \)

(g) \( \frac{3}{n^2 - 5n - 36} - \frac{2}{n^2 + 3n - 4} \)

(h) \( \frac{1}{x - 3} - \frac{3}{3 - x} \)

(i) \( \frac{1}{6x(x - 2)^2} - \frac{1}{4x^2(x - 2)} \)

2. Solve:

(a) \( \frac{1}{2x - 7} + \frac{x - 5}{4x^2 - 49} = \frac{4}{6x - 21} \)

(b) \( \frac{4}{5y - 3} = \frac{2}{3y + 7} \)

(c) \( n + \frac{1}{n} = \frac{53}{14} \)

(d) \( 2 + \frac{4}{x - 2} = \frac{8}{x^2 - 2x} \)

3. Simplify (express results with positive exponents only and rationalized denominators):

(a) \( 16^{-3/4} \)

(b) \( (4x^5y^{-2})^{-2} \)

(c) \( \frac{4\sqrt{12}}{\sqrt{5}} \)

(d) \( \sqrt{\frac{3}{8}} \)

(e) \( \sqrt[3]{\frac{27}{64}} \)

(f) \( \sqrt[5]{81x^3y^4z^7} \)

(g) \( (-8x^{-6}y^{12})^{1/3} \)

4. Perform the indicated operations and simplify:

(a) \( 3\sqrt{20} - \sqrt{5} - 3\sqrt{45} \)

(b) \( \sqrt{3} + 2 \left( \sqrt{24} \right) - 6 \left( \sqrt{81} \right) \)

(c) \( -4\sqrt{2} \left( 2\sqrt{5} - 3\sqrt{6} \right) \)

(d) \( \left( 2\sqrt{6} + 3\sqrt{5} \right) \left( \sqrt{8} - 3\sqrt{12} \right) \)

(e) \( \frac{\sqrt{7}}{3\sqrt{2} - 5} \)

(f) \( \frac{\sqrt{12}}{4} + \frac{\sqrt{27}}{6} \)
5. Solve for \( x \) and check your solutions:
   
   (a) \( \sqrt{2x - 5} = 5 \)   
   (b) \( \sqrt{4x + 2} = \sqrt{3x + 4} \)   
   (c) \( 4\sqrt{x} + 5 = x \)   
   (d) \( \sqrt{x + 1} - \sqrt{2x} = 1 \)

6. 
   (a) Solve by factoring: \( x^2 - 7x + 12 = 0 \)
   (b) Solve with the quadratic formula: \( 4x^2 + 2x + 1 = 0 \)
   (c) Solve by completing the square: \( 2x^2 - 8x - 3 = 0 \)
   (d) Solve by any method: \( x^2 + 10x + 26 = 0 \)

7. Simplify (express your results in the form \( a + bi \) for \( a, b \) real):
   
   (a) \( (4 - 8i) - (8 - 3i) \)   
   (b) \( \left(\sqrt{-4}\right)\left(\sqrt{-16}\right) \)   
   (c) \( 7i(-9 + 3i) \)   
   (d) \( (10 + 2i)(-2 - i) \)   
   (e) \( \frac{-1 - 3i}{4 - 5i} \)   
   (f) \( 2(\cos 120^\circ + i\sin 120^\circ) \)

8. Given \( f(x) = 3x^2 - 2 \), determine:
   
   (a) \( f(2) \)   
   (b) \( f(-3) \)   
   (c) \( f(1/2) \)   
   (d) \( f(a) \)

9. Sketch the graphs of each of the given functions, indicating the \( x \) and \( y \) intercepts, the vertex, the axis of symmetry and stating the maximum or minimum value of the function:

   (a) \( f(x) = x^2 - 3 \)   
   (b) \( g(x) = -2x^2 + 2 \)   
   (c) \( h(x) = -(x + 2)^2 \)   
   (d) \( k(x) = -(x + 2)^2 + 2 \)   
   (e) \( w(x) = x^2 + x + 1 \)

10. Given \( f(x) = \frac{x^2 - 9}{x^2 + 2x - 15} \)
    
    (a) Determine the values of \( x \) for which the function is defined
    (b) Evaluate: \( f(0) \)
    (c) Evaluate: \( f(-3) \)

11. Given the functions \( f(x) = 2x + 1 \) and \( g(x) = \frac{1}{2}x - \frac{1}{2} \) sketch both graphs on the same set of axes.

12. Sketch each pair of functions on the same set of axes:

   (a) \( f(x) = 3^x \) and \( g(x) = 3^x - 2 \)   
   (b) \( f(x) = 3^{-x} \) and \( g(x) = \log_3 x \)

13. Solve for \( x \) (use the definitions and properties of exponents and logarithms):

   (a) \( 3^{x-1} = 81 \)   
   (b) \( 7^{-x} = 49 \)   
   (c) \( 25^x = 125 \)

   (d) \( \log_4 1 = x \)   
   (e) \( \log_6 (6^{-8}) = x \)   
   (f) \( \log_2 x = -4 \)
14. (a) Find angle \( \theta \) in the second quadrant if \( \sin \theta = 1/2 \).
(b) Find angle \( \theta \) if \( \tan \theta = -1 \) and \( \cos \theta > 0 \).
(c) Find two angles \( \theta \) for which \( \cos \theta = -\sqrt{3}/2 \).

15. (a) Find \( \sin \theta \) if \( \cos \theta = -3/5 \) and \( \theta \) is in Quadrant III.
(b) Find \( \cot \theta \) if \( \tan \theta = 1/4 \).
(c) Find \( \sec \theta \) if \( \sin \theta = -1/4 \) and \( \cos \theta < 0 \).

16. Evaluate \( \sin \theta \), \( \cos \theta \) and \( \tan \theta \) exactly for each of the following angles:
   (a) \( \theta = 210^\circ \)  (b) \( \theta = -240^\circ \)  (c) \( \theta = 675^\circ \)

17. For each of the following angles \( \theta \), draw them in standard position, choose a specific point on the terminal side of \( \theta \) and determine the exact values of \( \sin \theta \), \( \cos \theta \) and \( \tan \theta \) without using a calculator:
   (a) \( \theta = \frac{5}{6} \pi \)  (b) \( \theta = 315^\circ \)  (c) \( \theta = 270^\circ \)

18. If each of the following points \( P \) are on the terminal side of angle \( \theta \) in standard position with \( 0 \leq \theta < 360^\circ \), draw \( \theta \) and determine the value of the six trigonometric functions of \( \theta \):
   (a) \( P = (3, -2) \)  (b) \( P = (-3, 4) \)  (c) \( P = (2, 4) \)

19. Solve the following (clearly specify the unknown, draw a labeled diagram if appropriate and state the solution in words):
   (a) The time a person takes to paddle a kayak 2 miles downstream is the same as the time to paddle half a mile upstream. If the rate of the current is 3 mph, what is the person’s paddling rate in still water?
   (b) Bill is standing on top of a 175 foot cliff overlooking a lake. The measure of the angle of depression to a boat is 30°. How far, exactly, is the boat from the bottom of the cliff?
   (c) Suppose that the height in meters of a golf ball, hit from a tee, is approximated by \( y = -5t^2 + 10t \) where \( t \) is the time in seconds. Find the maximum height of the ball and the time it reaches this maximum height.

20. Graph each equation for \(-2\pi \leq x \leq 2\pi\):
   (a) \( y = 2 \sin x \)  (b) \( y = 3 \cos x \)  (c) \( y = -\cos x \)

21. A central angle of \( \theta = 60^\circ \) is contained in a circle with radius \( r = 30 \) inches. Find (leaving all results in terms of \( \pi \)):
   (a) the length of the arc subtended by \( \theta \)
   (b) the area of the sector determined by \( \theta \)
22. Find the exact values of the area and perimeter of this right triangle:

\[ a = 2 \text{ in} \]

23. Verify the following trigonometric identities:
   (a) \( \csc \theta \tan \theta \cos \theta = 1 \)
   (b) \( \csc \theta - \sin \theta = \cot \theta \cos \theta \)
   (c) \( \tan^2 \theta + 1 = \sec^2 \theta \)
   (d) \( \cos^2 \theta - \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \)

24.
   (a) A hot-air balloon rises vertically. An observer stands on level ground at a distance of 125 feet from a point on the ground directly below the passenger’s compartment. How far, to the nearest foot, does the balloon rise if the angle of elevation changes from 30° to 45°?
   (b) A state trooper is hiding 30 feet from a straight highway with a speed limit of 65 mph. One second after a truck passes, the angle \( \theta \) between the highway and the line of observation from the patrol car to the truck is measured.
      (i) If \( \theta = 15° \) does the truck driver get a speeding ticket (1 mile = 5,280 ft)?
      (ii) If \( \theta = 30° \) does the truck driver get a speeding ticket?

Answers

1.
   (a) \( \frac{n - 5}{n - 1} \) (b) \( \frac{3xy + 5x}{3(3y + 5x)} \) (c) \( \frac{x}{x + 1} \) (d) \( \frac{x}{x + 3y} \) (e) \( \frac{9ab + 18b}{3a + b} \)
   (f) \( \frac{2}{x - 5} \) (g) \( \frac{n + 15}{(n - 9)(n + 4)(n - 1)} \) (h) \( \frac{4}{x - 3} \) (i) \( \frac{-x + 6}{12x^2(x - 2)^2} \)

2.
   (a) \( x = 22 \) (b) \( y = -17 \) (c) \( n = 2/7, 7/2 \) (d) \( x = -2 \)

3.
   (a) \( \frac{1}{8} \) (b) \( \frac{y^4}{16x^6} \) (c) \( \frac{8\sqrt{15}}{5} \) (d) \( \frac{\sqrt{6}}{4} \) (e) \( \frac{3}{4} \) (f) \( 3xyz^2 \sqrt{3yz} \) (g) \( \frac{-2y^4}{x^2} \)

4.
   (a) \( -4\sqrt{5} \) (b) \( -13\sqrt{3} \) (c) \( -8\sqrt{10} + 24\sqrt{3} \)
   (d) \( 8\sqrt{3} - 36\sqrt{2} + 6\sqrt{10} - 18\sqrt{15} \) (e) \( \frac{3\sqrt{14} + 5\sqrt{7}}{7} \) (f) \( \sqrt{3} \)
5.  
(a) $x = 15$  
(b) $x = 2$  
(c) $x = 25$ (1 is an extraneous solution)  
(d) $x = 0$ (8 is an extraneous solution)

6.  
(a) $x = 3, 4$  
(b) $x = \frac{-1 \pm i\sqrt{3}}{4}$  
(c) $x = 2 \pm \frac{\sqrt{22}}{2}$  
(d) $x = -5 \pm i$

7.  
(a) $-4 - 5i$  
(b) $-8$  
(c) $-21 - 63i$  
(d) $-18 - 14i$  
(e) $\frac{11}{41} - \frac{17}{41}i$  
(f) $-1 + i\sqrt{3}$

8.  
(a) $f(2) = 10$  
(b) $f(-3) = 25$  
(c) $f(1/2) = -5/4 = -1\frac{1}{4}$  
(d) $f(a) = 3a^2 - 2$

9.  
(a)  
Axis of symmetry $x = 0$  
Vertex $(0, -3)$  
Minimum $y = -3$  
y-intercept $(0, -3)$  
x-intercepts $(-\sqrt{3}, 0), (\sqrt{3}, 0)$

(b)  
Axis of symmetry $x = 0$  
Vertex $(0, 2)$  
Maximum $y = 2$  
y-intercept $(0, 2)$  
x-intercepts $(-1, 0), (1, 0)$

(c)  
Axis of symmetry $x = -2$  
Vertex $(-2, 0)$  
Maximum $y = 0$  
y-intercept $(0, -4)$  
x-intercept $(-2, 0)$

(d)  
Axis of symmetry $x = -2$  
Vertex $(-2, 2)$  
Maximum $y = 2$  
y-intercept $(0, -2)$  
x-intercepts $(-2 \pm \sqrt{2}, 0)$

(e)  
Axis of symmetry $x = -1/2$  
Vertex $(-1/2, 3/4)$  
Minimum $y = 3/4$  
y-intercept $(0, 1)$  
x-intercepts none

10.  
(a) $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$ or all real numbers except $-5$ and $3$  
(b) $f(0) = 3/5$  
(c) $f(-3) = 0$
11. 

![Graph of functions f(x) and g(x)]

12. 

(a) 

![Graph of functions f(x) and g(x)]

(b) 

![Graph of functions f(x) and g(x)]

13. 

(a) $x = 5$  
(b) $x = -2$  
(c) $x = \frac{3}{2}$  
(d) $x = 0$  
(e) $x = -8$  
(f) $x = \frac{1}{16}$

14. 

(a) $\theta = 150^\circ$  
(b) $\theta = 315^\circ$  
(c) $\theta = 150^\circ$ and $210^\circ$

15. 

(a) $\sin \theta = \frac{-4}{5}$  
(b) $\cot \theta = 4$  
(c) $\sec \theta = -\frac{4}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$

16. 

(a) $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$,  
$\cos 210^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$,  
$\tan 210^\circ = \tan 30^\circ = \frac{\sqrt{3}}{3}$  
(b) $\sin(-240^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$,  
$\cos(-240^\circ) = -\cos 60^\circ = -\frac{1}{2}$,  
$\tan(-240^\circ) = -\tan 60^\circ = -\sqrt{3}$  
(c) $\sin 675^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$,  
$\cos 675^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$,  
$\tan 675^\circ = -\tan 45^\circ = -1$
17.

(a) \( y \) \( x \) (3, -2)

\[
\sin \theta = \frac{-2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}
\]
\[
\cos \theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}
\]
\[
\tan \theta = \frac{-2}{3} = -\frac{2}{3}
\]
\[
\csc \theta = -\frac{\sqrt{13}}{2}
\]
\[
\sec \theta = \frac{\sqrt{13}}{3}
\]
\[
\cot \theta = \frac{3}{2}
\]

(b) (-3, 4) \( y \) \( x \)

\[
\sin \theta = \frac{4}{5}
\]
\[
\cos \theta = -\frac{3}{5}
\]
\[
\tan \theta = \frac{4}{-3} = -\frac{4}{3}
\]
\[
\csc \theta = \frac{5}{4}
\]
\[
\sec \theta = \frac{5}{3}
\]
\[
\cot \theta = -\frac{3}{4}
\]

(c) \( y \) \( x \) (2, 4)

\[
\sin \theta = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}
\]
\[
\cos \theta = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}
\]
\[
\tan \theta = \frac{4}{2} = 2
\]
\[
\csc \theta = \frac{\sqrt{5}}{2}
\]
\[
\sec \theta = \sqrt{5}
\]
\[
\cot \theta = \frac{1}{2}
\]
18.
(a) \((-\sqrt{3}, 1)\)  
(b) \((1, -1)\)  
(c) \((0, -1)\)

\[
\sin \frac{5\pi}{6} = \frac{1}{2} \\
\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} \\
\tan \frac{5\pi}{6} = -\frac{\sqrt{3}}{3}
\]

\[
\sin 315^\circ = -\frac{\sqrt{2}}{2} \\
\cos 315^\circ = \frac{\sqrt{2}}{2} \\
\tan 315^\circ = -1
\]

\[
\sin 270^\circ = -1 \\
\cos 270^\circ = 0 \\
\tan 270^\circ = \text{undefined}
\]

19.
(a) The person’s paddling rate in still water is 5 mph.
(b) The boat is \(175\sqrt{3}\) feet from the bottom of the cliff.
(c) The maximum height of the ball is 5 meters which it reaches in 1 second.

20.
(a) \(y = 2\sin x\)

(b) \(y = 3\cos x\)
21.  
(a) arc length = 10\pi inches  
(b) area = 150\pi square inches  

22.  
Area = 2 \tan 30^\circ \text{ in}^2 = \frac{2\sqrt{3}}{3} \text{ in}^2; \text{ perimeter} = 2\sqrt{3} + 2 \text{ in}  

23.  
To prove these identities, use algebra and the basic identities  
\[
\begin{align*}
\tan \theta &= \frac{\sin \theta}{\cos \theta} \\
\cos^2 \theta + \sin^2 \theta &= 1 \\
\csc \theta &= \frac{1}{\sin \theta} \\
\sec \theta &= \frac{1}{\cos \theta} \\
\cot \theta &= \frac{1}{\tan \theta}
\end{align*}
\]

24.  
(a) The balloon rises 53 feet.  
(b) (i) The truck is traveling at 111.96 ft/sec which is 76.34 mph and the driver gets a ticket. (ii) The truck is traveling at 51.96 ft/sec which is 35.43 mph and the driver does not get a speeding ticket.