

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

MTH06 Review Sheet

1. Perform the indicated operations and simplify:

(a) $\frac{n^2 - 3n - 10}{n^2 + n - 2}$

(b) $\frac{1 + \frac{5}{3y}}{\frac{6}{2x} + \frac{5}{y}}$

(c) $\frac{2x^2 - x}{2x^2 + x - 1}$

(d) $\left(\frac{x^2 - 2xy - 3y^2}{x^2 - 9y^2}\right) \left(\frac{2x^2 - xy}{2x^2 + xy - y^2}\right)$

(e) $\frac{9ab}{3a + b} \div \frac{a^2 - 6a}{a^2 - 4a - 12}$

(f) $\frac{10}{x^2 - 5x} + \frac{2}{x}$

(g) $\frac{3}{n^2 - 5n - 36} - \frac{2}{n^2 + 3n - 4}$

(h) $\frac{1}{x - 3} - \frac{3}{3 - x}$

(i) $\frac{1}{6x(x - 2)^2} - \frac{1}{4x^2(x - 2)}$

2. Solve:

(a) $\frac{1}{2x - 7} + \frac{x - 5}{4x^2 - 49} = \frac{4}{6x - 21}$

(b) $\frac{4}{5y - 3} = \frac{2}{3y + 7}$

(c) $n + \frac{1}{n} = \frac{53}{14}$

(d) $2 + \frac{4}{x - 2} = \frac{8}{x^2 - 2x}$

3. Simplify (express results with positive exponents only and rationalized denominators):

(a) $16^{-3/4}$

(b) $(4x^5y^{-2})^{-2}$

(c) $\frac{4\sqrt{12}}{\sqrt{5}}$

(d) $\sqrt{\frac{3}{8}}$

(e) $\sqrt[3]{\frac{27}{64}}$

(f) $\sqrt[3]{81x^3y^4z^7}$

(g) $(-8x^{-6}y^{12})^{1/3}$

4. Perform the indicated operations and simplify:

(a) $3\sqrt{20} - \sqrt{5} - 3\sqrt{45}$

(b) $\sqrt[3]{3} + 2(\sqrt[3]{24}) - 6(\sqrt[3]{81})$

(c) $-4\sqrt{2}(2\sqrt{5} - 3\sqrt{6})$

(d) $(2\sqrt{6} + 3\sqrt{5})(\sqrt{8} - 3\sqrt{12})$

(e) $\frac{\sqrt{7}}{3\sqrt{2} - 5}$

(f) $\frac{\sqrt{12}}{4} + \frac{\sqrt{27}}{6}$

5. Solve for x and check your solutions:

(a) $\sqrt{2x-5} = 5$ (b) $\sqrt{4x+2} = \sqrt{3x+4}$ (c) $4\sqrt{x} + 5 = x$ (d) $\sqrt{x+1} - \sqrt{2x} = 1$

6.

- (a) Solve by factoring: $x^2 - 7x + 12 = 0$
(b) Solve with the quadratic formula: $4x^2 + 2x + 1 = 0$
(c) Solve by completing the square: $2x^2 - 8x - 3 = 0$
(d) Solve by any method: $x^2 + 10x + 26 = 0$

7. Simplify (express your results in the form $a + bi$ for a, b real):

(a) $(4 - 8i) - (8 - 3i)$ (b) $(\sqrt{-4})(\sqrt{-16})$ (c) $7i(-9 + 3i)$
(d) $(10 + 2i)(-2 - i)$ (e) $\frac{-1 - 3i}{4 - 5i}$ (f) $2(\cos 120^\circ + i \sin 120^\circ)$

8. Given $f(x) = 3x^2 - 2$, determine:

(a) $f(2)$ (b) $f(-3)$ (c) $f(1/2)$ (d) $f(a)$

9. Sketch the graphs of each of the given functions, indicating the x and y intercepts, the vertex, the axis of symmetry and stating the maximum or minimum value of the function:

(a) $f(x) = x^2 - 3$ (b) $g(x) = -2x^2 + 2$ (c) $h(x) = -(x + 2)^2$
(d) $k(x) = -(x + 2)^2 + 2$ (e) $w(x) = x^2 + x + 1$

10. Given $f(x) = \frac{x^2 - 9}{x^2 + 2x - 15}$

- (a) Determine the values of x for which the function is defined
(b) Evaluate: $f(0)$
(c) Evaluate: $f(-3)$

11. Given the functions $f(x) = 2x + 1$ and $g(x) = \frac{1}{2}x - \frac{1}{2}$ sketch both graphs on the same set of axes.

12. Sketch each pair of functions on the same set of axes:

(a) $f(x) = 3^x$ and $g(x) = 3^x - 2$ (b) $f(x) = 3^{-x}$ and $g(x) = \log_3 x$

13. Solve for x (use the definitions and properties of exponents and logarithms):

(a) $3^{x-1} = 81$ (b) $7^{-x} = 49$ (c) $25^x = 125$
(d) $\log_4 1 = x$ (e) $\log_6(6^{-8}) = x$ (f) $\log_2 x = -4$

14.

- (a) Find angle θ in the second quadrant if $\sin \theta = 1/2$.
- (b) Find angle θ if $\tan \theta = -1$ and $\cos \theta > 0$.
- (c) Find two angles θ for which $\cos \theta = -\sqrt{3}/2$.

15.

- (a) Find $\sin \theta$ if $\cos \theta = -3/5$ and θ is in Quadrant III.
- (b) Find $\cot \theta$ if $\tan \theta = 1/4$.
- (c) Find $\sec \theta$ if $\sin \theta = -1/4$ and $\cos \theta < 0$.

16. Evaluate $\sin \theta$, $\cos \theta$ and $\tan \theta$ exactly for each of the following angles:

(a) $\theta = 210^\circ$ (b) $\theta = -240^\circ$ (c) $\theta = 675^\circ$

17. For each of the following angles θ , draw them in standard position, choose a specific point on the terminal side of θ and determine the exact values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ without using a calculator:

(a) $\theta = \frac{5}{6}\pi$ (b) $\theta = 315^\circ$ (c) $\theta = 270^\circ$

18. If each of the following points P are on the terminal side of angle θ in standard position with $0 \leq \theta < 360^\circ$, draw θ and determine the value of the six trigonometric functions of θ :

(a) $P = (3, -2)$ (b) $P = (-3, 4)$ (c) $P = (2, 4)$

19. Solve the following (clearly specify the unknown, draw a labeled diagram if appropriate and state the solution in words):

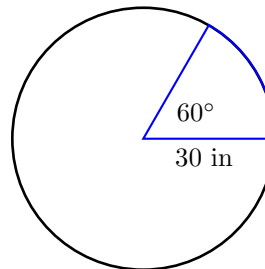
- (a) The time a person takes to paddle a kayak 2 miles downstream is the same as the time to paddle half a mile upstream. If the rate of the current is 3 mph, what is the person's paddling rate in still water?
- (b) Bill is standing on top of a 175 foot cliff overlooking a lake. The measure of the angle of depression to a boat is 30° . How far, exactly, is the boat from the bottom of the cliff?
- (c) Suppose that the height in meters of a golf ball, hit from a tee, is approximated by $y = -5t^2 + 10t$ where t is the time in seconds. Find the maximum height of the ball and the time it reaches this maximum height.

20. Graph each equation for $-2\pi \leq x \leq 2\pi$:

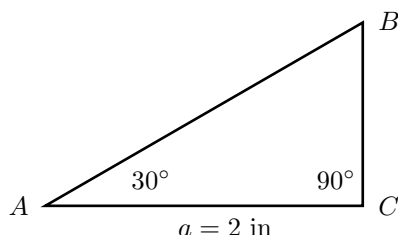
(a) $y = 2 \sin x$ (b) $y = 3 \cos x$ (c) $y = -\cos x$

21. A central angle of $\theta = 60^\circ$ is contained in a circle with radius $r = 30$ inches. Find (leaving all results in terms of π):

- (a) the length of the arc subtended by θ
- (b) the area of the sector determined by θ



22. Find the exact values of the area and perimeter of this right triangle:



23. Verify the following trigonometric identities:

- (a) $\csc \theta \tan \theta \cos \theta = 1$
 (b) $\csc \theta - \sin \theta = \cot \theta \cos \theta$
 (c) $\tan^2 \theta + 1 = \sec^2 \theta$
 (d) $\cos^2 \theta - \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

24.

- (a) A hot-air balloon rises vertically. An observer stands on level ground at a distance of 125 feet from a point on the ground directly below the passenger's compartment. How far, to the nearest foot, does the balloon rise if the angle of elevation changes from 30° to 45° ?
- (b) A state trooper is hiding 30 feet from a straight highway with a speed limit of 65 mph. One second after a truck passes, the angle θ between the highway and the line of observation from the patrol car to the truck is measured.
- (i) If $\theta = 15^\circ$ does the truck driver get a speeding ticket (1 mile = 5,280 ft)?
 (ii) If $\theta = 30^\circ$ does the truck driver get a speeding ticket?

Answers

1.

- (a) $\frac{n-5}{n-1}$ (b) $\frac{3xy+5x}{3(3y+5x)}$ (c) $\frac{x}{x+1}$ (d) $\frac{x}{x+3y}$ (e) $\frac{9ab+18b}{3a+b}$
 (f) $\frac{2}{x-5}$ (g) $\frac{n+15}{(n-9)(n+4)(n-1)}$ (h) $\frac{4}{x-3}$ (i) $\frac{-x+6}{12x^2(x-2)^2}$

2.

- (a) $x = 22$ (b) $y = -17$ (c) $n = 2/7, 7/2$ (d) $x = -2$

3.

- (a) $\frac{1}{8}$ (b) $\frac{y^4}{16x^{10}}$ (c) $\frac{8\sqrt{15}}{5}$ (d) $\frac{\sqrt{6}}{4}$ (e) $\frac{3}{4}$ (f) $3xyz^2\sqrt[3]{3yz}$ (g) $\frac{-2y^4}{x^2}$

4.

- (a) $-4\sqrt{5}$ (b) $-13\sqrt[3]{3}$ (c) $-8\sqrt{10} + 24\sqrt{3}$
 (d) $8\sqrt{3} - 36\sqrt{2} + 6\sqrt{10} - 18\sqrt{15}$ (e) $-\frac{3\sqrt{14} + 5\sqrt{7}}{7}$ (f) $\sqrt{3}$

5.

- (a) $x = 15$ (b) $x = 2$ (c) $x = 25$ (1 is an extraneous solution) (d) $x = 0$ (8 is an extraneous solution)

6.

- (a) $x = 3, 4$ (b) $x = \frac{-1 \pm i\sqrt{3}}{4}$ (c) $x = 2 \pm \frac{\sqrt{22}}{2}$ (d) $x = -5 \pm i$

7.

- (a) $-4 - 5i$ (b) -8 (c) $-21 - 63i$ (d) $-18 - 14i$ (e) $\frac{11}{41} - \frac{17}{41}i$ (f) $-1 + i\sqrt{3}$

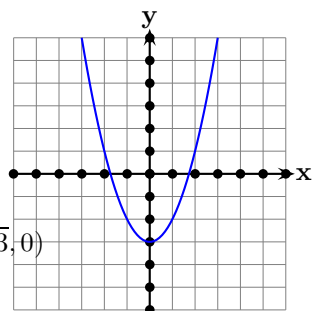
8.

- (a) $f(2) = 10$ (b) $f(-3) = 25$ (c) $f(1/2) = -5/4 = -1\frac{1}{4}$ (d) $f(a) = 3a^2 - 2$

9.

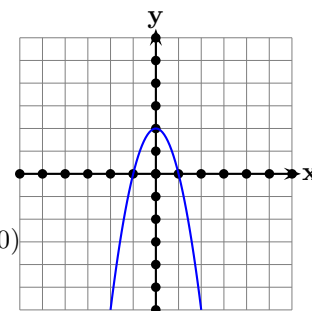
(a)

- Axis of symmetry $x = 0$
 Vertex $(0, -3)$
 Minimum $y = -3$
 y -intercept $(0, -3)$
 x -intercepts $(-\sqrt{3}, 0), (\sqrt{3}, 0)$



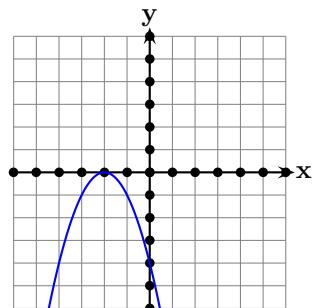
(b)

- Axis of symmetry $x = 0$
 Vertex $(0, 2)$
 Maximum $y = 2$
 y -intercept $(0, 2)$
 x -intercepts $(-1, 0), (1, 0)$



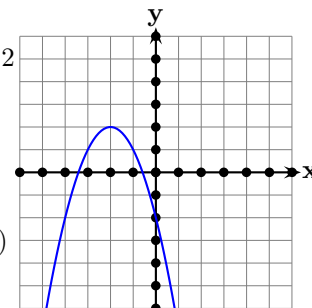
(c)

- Axis of symmetry $x = -2$
 Vertex $(-2, 0)$
 Maximum $y = 0$
 y -intercept $(0, -4)$
 x -intercept $(-2, 0)$



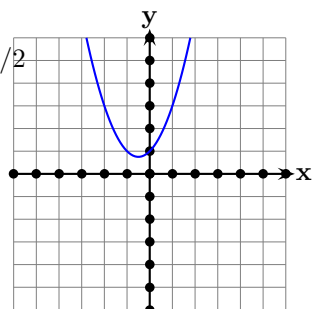
(d)

- Axis of symmetry $x = -2$
 Vertex $(-2, 2)$
 Maximum $y = 2$
 y -intercept $(0, -2)$
 x -intercepts $(-2 \pm \sqrt{2}, 0)$



(e)

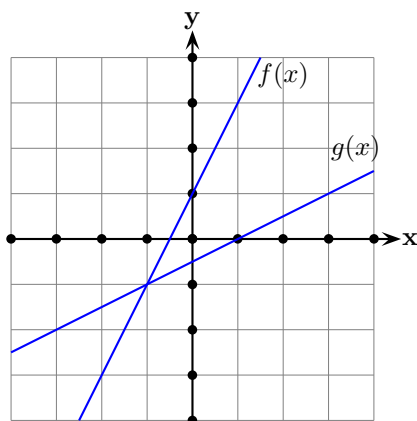
- Axis of symmetry $x = -1/2$
 Vertex $(-1/2, 3/4)$
 Minimum $y = 3/4$
 y -intercept $(0, 1)$
 x -intercepts none



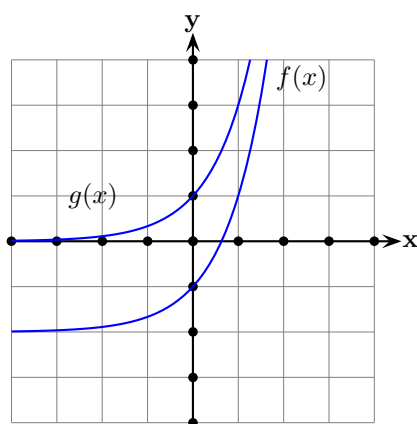
10.

- (a) $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$ or all real numbers except -5 and 3 (b) $f(0) = 3/5$ (c) $f(-3) = 0$

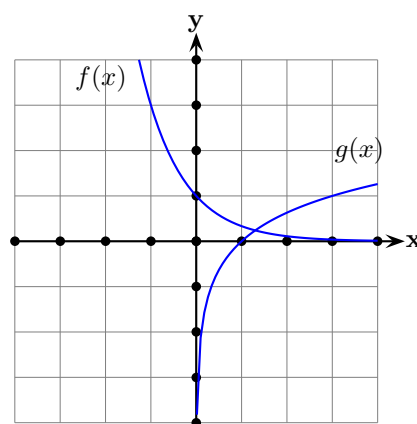
11.



12.



(a)



(b)

13.

- (a) $x = 5$ (b) $x = -2$ (c) $x = 3/2$ (d) $x = 0$ (e) $x = -8$ (f) $x = 1/16$

14.

- (a) $\theta = 150^\circ$ (b) $\theta = 315^\circ$ (c) $\theta = 150^\circ$ and 210°

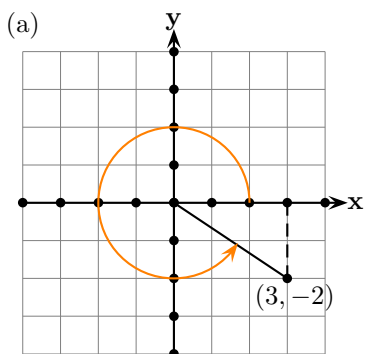
15.

- (a) $\sin \theta = -4/5$ (b) $\cot \theta = 4$ (c) $\sec \theta = -4/\sqrt{15} = -4\sqrt{15}/15$

16.

- (a) $\sin 210^\circ = -\sin 30^\circ = -1/2$, $\cos 210^\circ = -\cos 30^\circ = -\sqrt{3}/2$, $\tan 210^\circ = \tan 30^\circ = \sqrt{3}/3$
 (b) $\sin(-240^\circ) = \sin 60^\circ = \sqrt{3}/2$, $\cos(-240^\circ) = -\cos 60^\circ = -1/2$, $\tan(-240^\circ) = -\tan 60^\circ = -\sqrt{3}$
 (c) $\sin 675^\circ = -\sin 45^\circ = -\sqrt{2}/2$, $\cos 675^\circ = \cos 45^\circ = \sqrt{2}/2$, $\tan 675^\circ = -\tan 45^\circ = -1$

17.



$$\sin \theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$$

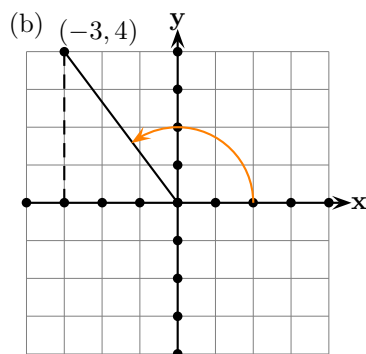
$$\cos \theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\tan \theta = \frac{-2}{3} = -\frac{2}{3}$$

$$\csc \theta = -\frac{\sqrt{13}}{2}$$

$$\sec \theta = \frac{\sqrt{13}}{3}$$

$$\cot \theta = -\frac{3}{2}$$



$$\sin \theta = \frac{4}{5}$$

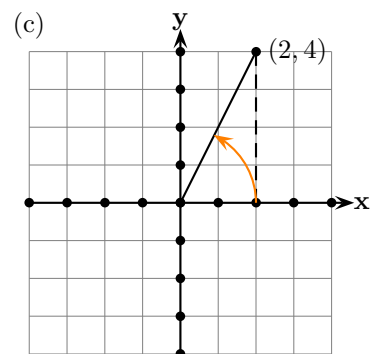
$$\cos \theta = \frac{-3}{5}$$

$$\tan \theta = \frac{4}{-3} = -\frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = -\frac{5}{3}$$

$$\cot \theta = -\frac{3}{4}$$



$$\sin \theta = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$$

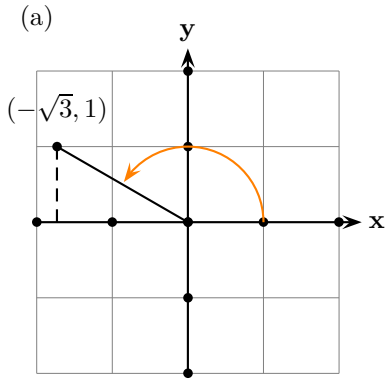
$$\tan \theta = \frac{4}{2} = 2$$

$$\csc \theta = \frac{\sqrt{5}}{2}$$

$$\sec \theta = \sqrt{5}$$

$$\cot \theta = \frac{1}{2}$$

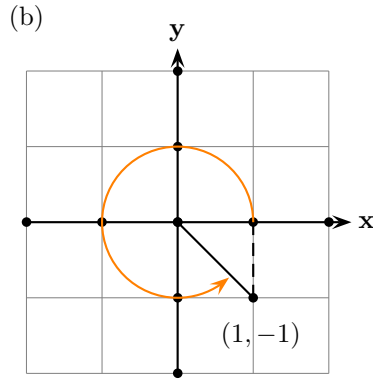
18.



$$\sin \frac{5}{6}\pi = \frac{1}{2}$$

$$\cos \frac{5}{6}\pi = \frac{-\sqrt{3}}{2}$$

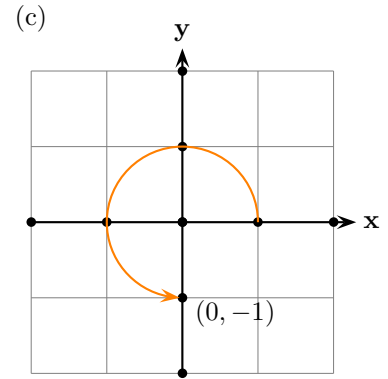
$$\tan \frac{5}{6}\pi = \frac{1}{-\sqrt{3}} = \frac{-\sqrt{3}}{3}$$



$$\sin 315^\circ = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$\cos 315^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 315^\circ = \frac{-1}{1} = -1$$



$$\sin 270^\circ = \frac{-1}{1} = -1$$

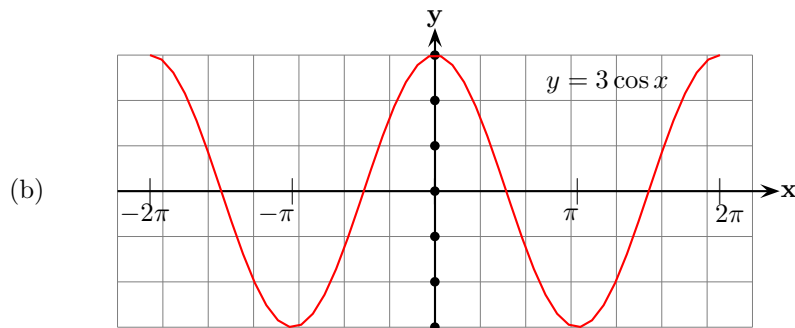
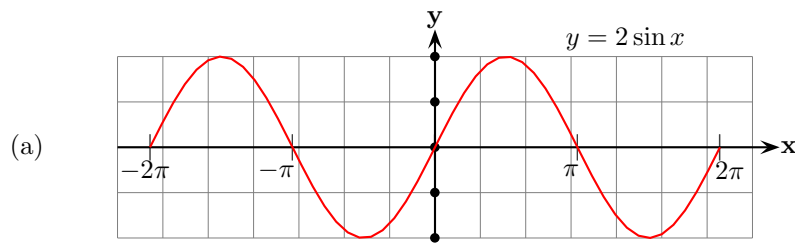
$$\cos 270^\circ = \frac{0}{1} = 0$$

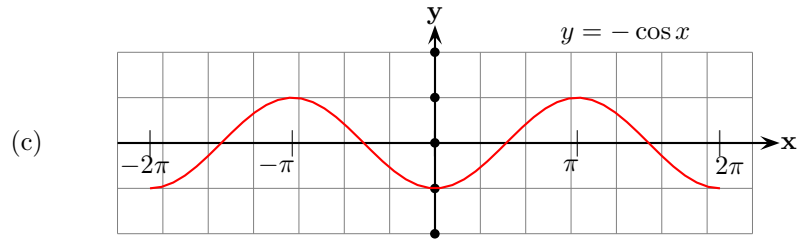
$$\tan 270^\circ = \frac{-1}{0}, \text{ undefined}$$

19.

- (a) The person's paddling rate in still water is 5 mph.
- (b) The boat is $175\sqrt{3}$ feet from the bottom of the cliff.
- (c) The maximum height of the ball is 5 meters which it reaches in 1 second.

20.





21.

- (a) arc length = 10π inches (b) area = 150π square inches

22.

$$\text{Area} = 2 \tan 30^\circ \text{ in}^2 = \frac{2\sqrt{3}}{3} \text{ in}^2; \text{ perimeter} = 2\sqrt{3} + 2 \text{ in}$$

23.

To prove these identities, use algebra and the basic identities

$$\begin{aligned} \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \cos^2 \theta + \sin^2 \theta &= 1 \\ \csc \theta &= \frac{1}{\sin \theta} \\ \sec \theta &= \frac{1}{\cos \theta} \\ \cot \theta &= \frac{1}{\tan \theta} \end{aligned}$$

24.

- (a) The balloon rises 53 feet.
- (b) (i) The truck is traveling at 111.96 ft/sec which is 76.34 mph and the driver gets a ticket. (ii) The truck is traveling at 51.96 ft/sec which is 35.43 mph and the driver does not get a speeding ticket.