

**WORKBOOK. MTH 01 - FUNDAMENTAL CONCEPTS AND SKILLS IN
ARITHMETIC AND ALGEBRA.**

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

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CONTENTS

1. Addition, Subtraction, and Multiplication of Whole numbers	3
2. Powers and divisions of Whole numbers	6
3. Square roots, Order of operations, Average, and Geometry	8
4. Meaning of Fractions, Multiplications of Fractions, and Mixed numbers	13
5. Equivalent fractions, prime factorization, reducing fractions, shortened multiplications	17
6. Adding, subtracting like fractions, Lowest Common Denominator, Adding and Subtracting Unlike fractions, and Adding mixed numbers	20
7. Subtracting, multiplying mixed numbers and division of fractions	23
8. Combined operations and comparing fractions	27
9. Reading, Writing, Rounding off, adding and subtracting decimals	29
10. Multiplication, division of decimals by decimals and powers of ten	33
11. Combined operations, changing fractions to decimals, and decimals to fractions, comparing decimals	35
12. Ratio, Rate, and Proportion problems	37
13. Word problems using proportions and similar triangles	40
14. Percentages	43
15. Finding a fractional part of a number, percent problems and word problems using percent	45
16. Signed numbers and adding signed numbers	48
17. Subtracting and multiplying signed numbers	54
18. Dividing signed numbers, properties, operations with zero	58
19. Powers and square roots of signed numbers	64
20. Scientific Notation	69
21. Order of operations, and grouping symbols	70
22. Evaluating expressions and formulas	73
23. Solving linear equations	78
24. Practice problems	81

1. ADDITION, SUBTRACTION, AND MULTIPLICATION OF WHOLE NUMBERS

(1) What is a sum? What are the addends? Give five examples.

(2) State the commutative property of addition. Explain using an example.

(3) State the associative property of addition. Explain using an example.

(4) What is the additive identity? Explain its property.

(5) Find the sums:

509	35	394	1,432	2,040
<u>+ 785</u>	<u>+ 789</u>	+ 48	+1,345	+ 3,050
		<u>+ 136</u>	<u>+ 530</u>	+ 38
				<u>+ 2,327</u>

(6) Arrange the numbers in a vertical column and add:

(a) $98 + 432 + 1,349 + 12,908 + 132,752 + 8,234,654$.

(b) $2,089 + 78 + 198 + 200,580 + 900$

- (7) Rob is 16 years old, Tom is 32 years old, and Jerry is 5 years older than the combined ages of Rob and Tom. Find the combined ages of the three.
- (8) What is a difference of two numbers? What are the minuend and the subtrahend? Explain with an example.
- (9) Is subtraction commutative? Explain with an example.
- (10) Is subtraction associative? Explain with an example.
- (11) Subtract 3,490 from 48,531.
- (12) Subtract 478 from 8,230.
- (13) Find the differences:
- | | | | | |
|-------------|----------------|-------------|----------------|--------------|
| 345 | 4,321 | 125 | 80,000 | 3,205 |
| - <u>39</u> | - <u>1,987</u> | - <u>70</u> | - <u>7,896</u> | - <u>123</u> |
- (14) An apartment was listed for \$ 289,670. I can pay \$ 23,472 in cash. How much money do I need to borrow to buy that apartment?

(15) Find the products:

$$\begin{array}{r} 238 \\ \times 7 \\ \hline \end{array} \quad \begin{array}{r} 621 \\ \times 6 \\ \hline \end{array} \quad \begin{array}{r} 709 \\ \times 9 \\ \hline \end{array} \quad \begin{array}{r} 435 \\ \times 8 \\ \hline \end{array} \quad \begin{array}{r} 9800 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 238 \\ \times 24 \\ \hline \end{array} \quad \begin{array}{r} 621 \\ \times 317 \\ \hline \end{array} \quad \begin{array}{r} 709 \\ \times 405 \\ \hline \end{array} \quad \begin{array}{r} 435 \\ \times 170 \\ \hline \end{array} \quad \begin{array}{r} 9800 \\ \times 3200 \\ \hline \end{array}$$

$$3,205 \cdot 239$$

$$3,598 \cdot 7,823$$

$$32,234 \cdot 1,238$$

$$43,500 \cdot 27,000$$

(16) A town has 45,281 families. Each family pays on an average of \$ 1,342 in property taxes.

What is the town's income in property taxes?

2. POWERS AND DIVISIONS OF WHOLE NUMBERS

(1) In the expression 2^5 , the number 2 is the **base** and the number 5 is the **exponent**. Identify the base and the exponent in the following expressions:

(a) $3^4 =$ _____. Here the base is _____ and the exponent is _____.

(b) $5^4 =$ _____. Here the base is _____ and the exponent is _____.

(c) $4^3 =$ _____. Here the base is _____ and the exponent is _____.

(d) $7^3 =$ _____. Here the base is _____ and the exponent is _____.

(2) Rewrite each expression using exponents and find its value:

(a) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 =$ _____.

(b) $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 =$ _____.

(c) $10 \cdot 10 \cdot 10 \cdot 10 =$ _____.

(d) $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 =$ _____.

(3) Evaluate the following:

(a) 2^5

(b) 2^0

(c) 0^0

(d) 0^2

(e) $23 \cdot 10^7$

(f) $12,435 \cdot 10^2$

(g) $765 \cdot 10^5$

(4) Complete the following table of squares:

$0^2 =$	$6^2 =$	$12^2 =$	$18^2 =$
$1^2 =$	$7^2 =$	$13^2 =$	$19^2 =$
$2^2 =$	$8^2 =$	$14^2 =$	$20^2 =$
$3^2 =$	$9^2 =$	$15^2 =$	$30^2 =$
$4^2 =$	$10^2 =$	$16^2 =$	$40^2 =$
$5^2 =$	$11^2 =$	$17^2 =$	$50^2 =$

(5) Complete the following table of cubes:

$0^3 =$	$3^3 =$	$6^3 =$	$9^3 =$
$1^3 =$	$4^3 =$	$7^3 =$	$10^3 =$
$2^3 =$	$5^3 =$	$8^3 =$	$100^3 =$

(6) Work each short division and check your work:

$2,345 \div 7$

$3,264 \div 8$

$2,001 \div 6$

$2,745 \div 9$

$8,420 \div 5$

(7) Work each long division and check your work:

$135,621 \div 27$

$26,676 \div 38$

$28,224 \div 56$

$8,268 \div 39$

$144,200 \div 25$

(8) Work each long division and check your work:

$834,624 \div 207$

$668,754 \div 318$

$152,962 \div 526$

$237,978 \div 339$

$788,655 \div 245$

(9) Rob's annual salary is \$ 23,054. What is his monthly salary? What is his biweekly salary?
What is his weekly salary?

3. SQUARE ROOTS, ORDER OF OPERATIONS, AVERAGE, AND GEOMETRY

(1) Find the indicated square roots:

(a) $\sqrt{25}$

(b) $\sqrt{64}$

(c) $\sqrt{169}$

(d) $\sqrt{100}$

(e) $\sqrt{10,000}$

(f) $\sqrt{1}$

(2) Complete the following table of square roots:

$\sqrt{0} =$	$= 6$	$= 12$	$= 18$
$\sqrt{1} =$	$= 7$	$= 13$	$= 19$
$\sqrt{4} =$	$= 8$	$= 14$	$= 20$
$\sqrt{9} =$	$= 9$	$= 15$	$= 30$
$= 4$	$= 10$	$= 16$	$= 40$
$= 5$	$= 11$	$= 17$	$= 50$

(3) Complete the following table of cube roots:

$\sqrt[3]{0} =$	$= 3$	$= 6$	$= 9$
$\sqrt[3]{1} =$	$= 4$	$= 7$	$= 10$
$\sqrt[3]{8} =$	$= 5$	$= 8$	$= 100$

The **order of operations** are:

- Simplify parentheses or any groupings (like complicated square roots) first;
- Powers and roots (exponents) next;
- Multiplications and divisions (left to right) next;
- Additions and subtractions (left to right) last.

To illustrate, simplify the following:

(1) $3 \times 5 + 4$

(2) $3 \times (5 + 4)$

(3) $3 \times 5 - 4$

(4) $3 \times (5 - 4)$

$$(5) 12 \div 6 \div 2$$

$$(6) 12 \div (6 \div 2)$$

$$(7) 12 \times 6 \div 2$$

$$(8) 12 \div 6 \times 2$$

$$(9) 3 \cdot 4^2$$

$$(10) (3 \cdot 4)^2$$

$$(11) 32 - (16 \div 8 \times 2 + 2^4)$$

$$(12) (7 - 4)^2 + 3 \times 4 \div 2 - 12$$

$$(13) 3 \times 2 \times 5 \times 6 \div 10 - \sqrt{4 + 3 \times 2 + 4^2 + 5 \times 2}$$

Find the average of the following numbers:

$$(1) 20, 21, 23, 27, 29, 30$$

$$(2) 28, 30, 35, 40, 42$$

(3) 72, 75, 77, 80

(4) 110, 113, 115, 117, 120

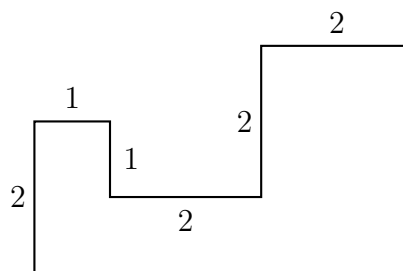
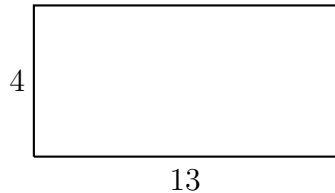
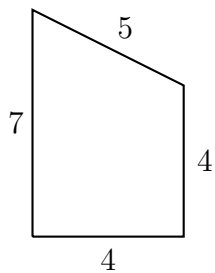
(5) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

A student scores 78, 80, 82, 84 in the four tests of her class. What is her average score in the course.

A student scores 72, 81, 78 in the first three tests of her class. What should be her score in the fourth test if the average is to be 80.

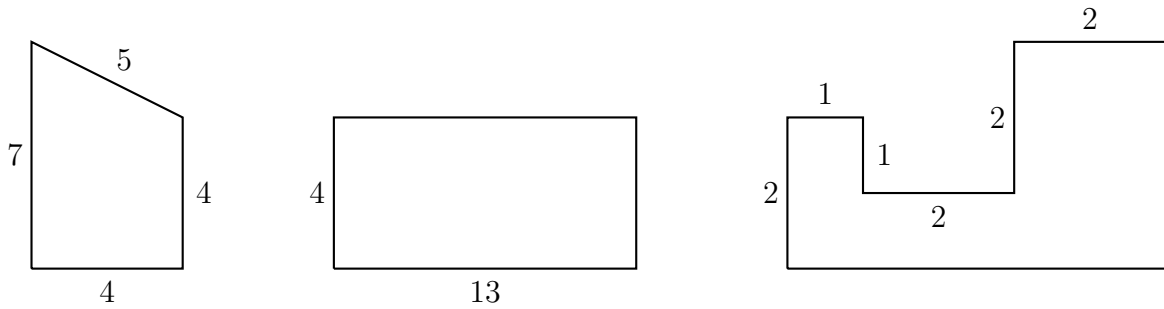
A student scores 82, 93, 88, 89 in the first four tests of her class. What should be her score in the fifth test if the average is to be 90.

Find the perimeter of each figure (assume that the units are in feet):



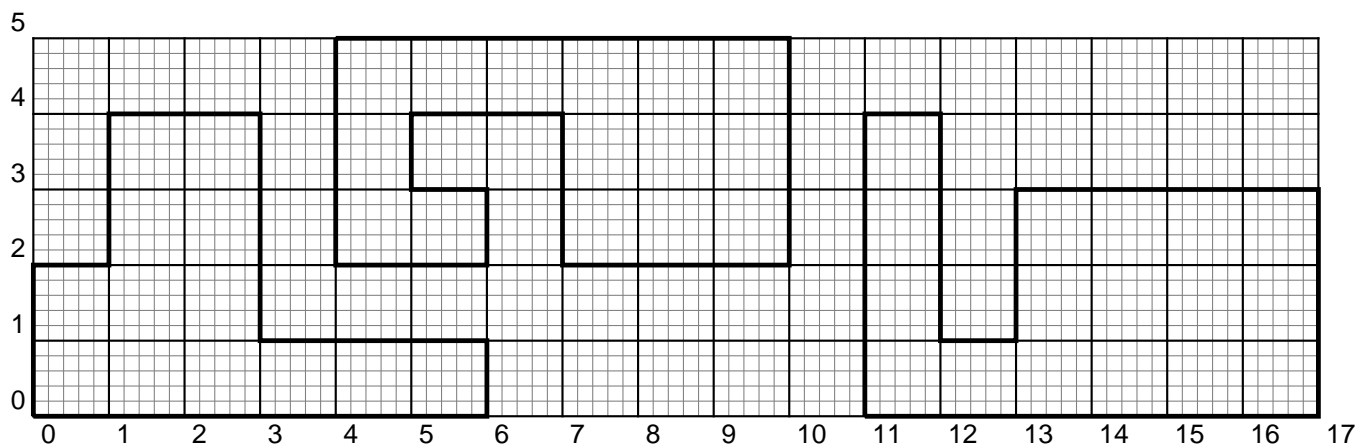
In each of the above figures, what will be the cost of fencing those figures if cost of the fencing material is \$6 per foot?

Find the area of each figure (assume that the units are in feet):



In each of the above figures, what will be the cost of carpeting those figures if cost of the carpet is \$ 12 per square foot?

Find the perimeter and area of each of these figures. Assume that the lengths are given in centimeters.



4. MEANING OF FRACTIONS, MULTIPLICATIONS OF FRACTIONS, AND MIXED NUMBERS

(1) How is a fraction written? What is the numerator and what is the denominator? What is the restriction on the denominator?

(2) Using rectangles, shade the parts representing $\frac{1}{2}$, $\frac{1}{5}$, $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{14}{7}$.

(3) Using circles, shade the parts representing $\frac{1}{3}$, $\frac{3}{6}$, $\frac{1}{4}$, $\frac{6}{3}$, and $\frac{3}{2}$.

(4) If we take a whole and divide it into eight pieces and take three, then what part of the whole is not taken?

(5) What are proper fractions? Give five examples. What are the numerators and denominators of each of the fractions?

(6) Write five fractions which are equal to 1 in value.

(7) How is a whole number written in the form of a fraction? Give five examples.

(8) Using five examples show how fractions are multiplied.

(9) Multiply the following:

(a) $\frac{3}{4} \cdot \frac{1}{8}$

(b) $\frac{5}{7} \cdot 3$

(c) $8 \cdot \frac{7}{9}$

(d) $\frac{3}{4} \cdot \frac{3}{4}$

(e) $\left(\frac{2}{5}\right)^2$

(f) $\left(\frac{5}{3}\right)^3$

(g) $\frac{2}{7} \cdot \frac{6}{5} \cdot 4$

(10) What is an improper fraction? Give five examples.

(11) What is a mixed fraction? Give five examples.

(12) Is $2 \cdot \frac{1}{3}$ a mixed fraction? Explain

(13) Convert the five examples of improper fractions you gave above into mixed fractions.

(14) Convert the following improper fractions into mixed fractions.

(a) $\frac{4}{3}$

(b) $\frac{40}{13}$

(c) $\frac{100}{9}$

(d) $\frac{101}{8}$

(e) $\frac{237}{12}$

(15) Explain using circles the improper fraction $\frac{13}{4}$. Do you see why $\frac{13}{4} = 3\frac{1}{4}$?

(16) Convert the mixed fractions to improper fractions.

(a) $3\frac{1}{7}$

(b) $5\frac{2}{8}$

(c) $4\frac{8}{12}$

(d) $5\frac{9}{13}$

(e) $7\frac{2}{15}$

(f) $4\frac{2}{103}$

5. EQUIVALENT FRACTIONS, PRIME FACTORIZATION, REDUCING FRACTIONS, SHORTENED
MULTIPLICATIONS

(1) What are equivalent fractions? Give five pairs of equivalent fractions.

(2) Explain using a rectangle why $\frac{3}{5} = \frac{6}{10}$.

(3) Fill in the blanks:

(a) $\frac{12}{18} = \frac{\quad}{9} = \frac{4}{\quad} = \frac{\quad}{3}$

(b) $\frac{100}{150} = \frac{\quad}{15} = \frac{20}{\quad} = \frac{\quad}{3}$

(c) $\frac{96}{120} = \frac{\quad}{10} = \frac{48}{\quad} = \frac{\quad}{5}$

(4) What is a prime number? Give five examples.

(5) What is a composite number? Give five examples.

(6) What is the divisibility test for 2? Give five examples of six-digit numbers divisible by 2.

(7) What is the divisibility test for 3? Give five examples of six-digit numbers divisible by 3.

(8) What is the divisibility test for 5? Give five examples of six-digit numbers divisible by 5.

(9) Prime factor the following:

(a) 12

(b) 25

(c) 48

(d) 100

(e) 120

(f) 390

(g) 54

(h) 360

(i) 288

(j) 234

(10) Reduce each fraction to its lowest terms:

(a) $\frac{30}{45}$

(b) $\frac{2000}{6400}$

(c) $\frac{155}{145}$

(d) $\frac{1017}{300}$

(e) $\frac{2500}{45000}$

(11) Multiply the following fractions and reduce them to their lowest form. Convert any resulting improper fractions to mixed numbers.

(a) $\frac{2}{15} \cdot \frac{5}{18}$

(b) $\frac{7}{66} \cdot \frac{18}{49} \cdot \frac{2}{15}$

(c) $\frac{3}{40} \cdot \frac{20}{39} \cdot \frac{13}{14}$

(d) $\frac{5}{36} \cdot \frac{18}{125} \cdot \frac{8}{27}$

(e) $\frac{7}{12} \cdot \frac{25}{48} \cdot \frac{2}{15} \cdot 3$

(12) Find:

(a) $\frac{3}{4}$ of 20.

(b) $\frac{3}{5}$ of 20.

(c) $\frac{2}{7}$ of 28.

(13) Four-fifths of the cars on the street lot are legally parked. There are 50 cars on the street right now. How many cars are legally parked? How many are not legally parked?

6. ADDING, SUBTRACTING LIKE FRACTIONS, LOWEST COMMON DENOMINATOR, ADDING AND
SUBTRACTING UNLIKE FRACTIONS, AND ADDING MIXED NUMBERS

(1) Add or subtract the following like fractions. Reduce your final answers and convert any of the improper results to mixed numbers:

(a) $\frac{3}{4} + \frac{1}{4}$

(b) $\frac{5}{12} + \frac{7}{12} - \frac{11}{12} + \frac{7}{12}$

(c) $\frac{2}{13} + \frac{7}{13} + \frac{8}{13} - \frac{12}{13}$

(d) $\frac{23}{100} - \frac{13}{100} - \frac{7}{100} + \frac{97}{100}$

(e) $\frac{9}{20} + \frac{7}{20} + \frac{14}{20} - \frac{1}{20}$

(2) Assume that the sets represent denominators of fractions. Find the LCD by inspection:

(a) {12, 18, 6}

(b) {10, 20, 40, 30}

(c) {6, 33, 22}

(d) {20, 45, 12}

(e) {8, 6, 12, 3}

(3) Assume that the sets represent denominators of fractions. Find the LCD by prime factorization:

(a) {20, 30, 60, 50, 100}

(b) {12, 8, 6}

(c) {1000, 3, 2}

(d) $\{18, 9, 12\}$

(e) $\{12, 3, 8, 18\}$

(4) Add or subtract these unlike fractions. Reduce the result to its lowest terms. If the resulting fraction is improper, then convert it to a mixed number.

(a) $\frac{11}{12} + \frac{5}{18} - \frac{1}{6}$

(b) $\frac{1}{6} - \frac{4}{33} + \frac{2}{11} - \frac{1}{22}$

(c) $\frac{3}{4} + \frac{7}{30} - \frac{2}{15}$

(d) $\frac{9}{20} - \frac{5}{8} + \frac{7}{10}$

(e) $\frac{3}{4} - \frac{1}{50} - \frac{7}{25}$

(5) Add these mixed numbers.

(a) $3\frac{1}{5} + 2\frac{2}{7}$

(b) $2\frac{7}{12} + 3\frac{5}{18}$

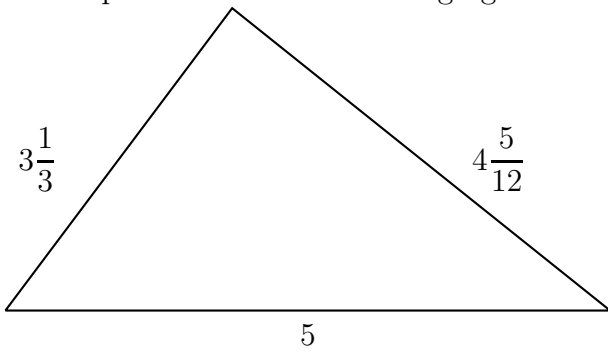
(c) $1\frac{7}{30} + 3\frac{2}{15} + 7\frac{3}{4}$

(d) $11\frac{2}{5} + 1\frac{3}{20} + 3$

(e) $5\frac{3}{4} + 7\frac{5}{8}$

(f) $4\frac{7}{9} + 1\frac{6}{7}$

(6) Find the perimeter of the following figure:



7. SUBTRACTING, MULTIPLYING MIXED NUMBERS AND DIVISION OF FRACTIONS

(1) Find the result. Reduce the answers to the lowest terms. Convert the improper fractions to mixed fractions.

(a) $7\frac{3}{4} - 5\frac{2}{5}$

(b) $7\frac{2}{5} - 5\frac{3}{4}$

(c) $7\frac{11}{15} - 7\frac{7}{10}$

(d) $7\frac{7}{10} - 7\frac{11}{15}$

(e) $4\frac{3}{7} + 2\frac{5}{11} - 2$

(f) $6\frac{1}{12} - 3\frac{7}{9} - 1\frac{1}{3}$

(g) $2\frac{3}{13} + \frac{5}{39} - 1\frac{1}{3}$

(2) From a bag containing $5\frac{2}{3}$ pounds of flour, the chef removed $2\frac{1}{7}$ pounds of flour. How much flour is left in the bag?

(3) Multiply the mixed numbers. Reduce the answers to the lowest terms. Convert the improper fractions to mixed fractions.

(a) $2\frac{1}{3} \cdot 4\frac{2}{5}$

(b) $3\frac{2}{7} \cdot \frac{1}{3}$

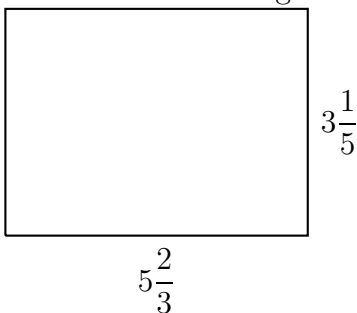
(c) $2\frac{4}{5} \cdot 1\frac{3}{4} \cdot 2$

(d) $1\frac{2}{3} \cdot 2\frac{1}{4} \cdot \frac{3}{8}$

(e) $2\frac{2}{5} \cdot 4\frac{1}{4} \cdot 4$

- (4) Two-third of a bin is full of flour. If the bin can carry 17 pounds of flour, then how much flour is in the bin?

- (5) Find the area of the figure:



- (6) Find the quotients. Reduce the answers to the lowest terms. Convert the improper fractions to mixed fractions.

(a) $\frac{3}{4} \div \frac{5}{16}$

(b) $\frac{7}{15} \div \frac{6}{25}$

(c) $10 \div \frac{5}{12}$

(d) $\frac{3}{4} \div 2$

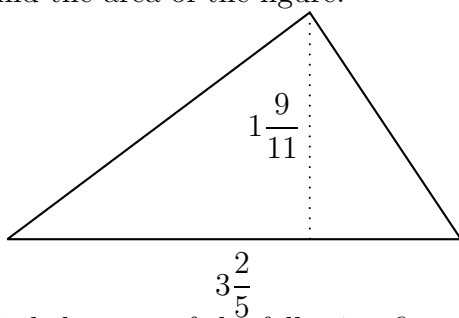
(e) $4\frac{3}{4} \div 5\frac{5}{12}$

(f) $11\frac{1}{4} \div 7\frac{2}{3}$

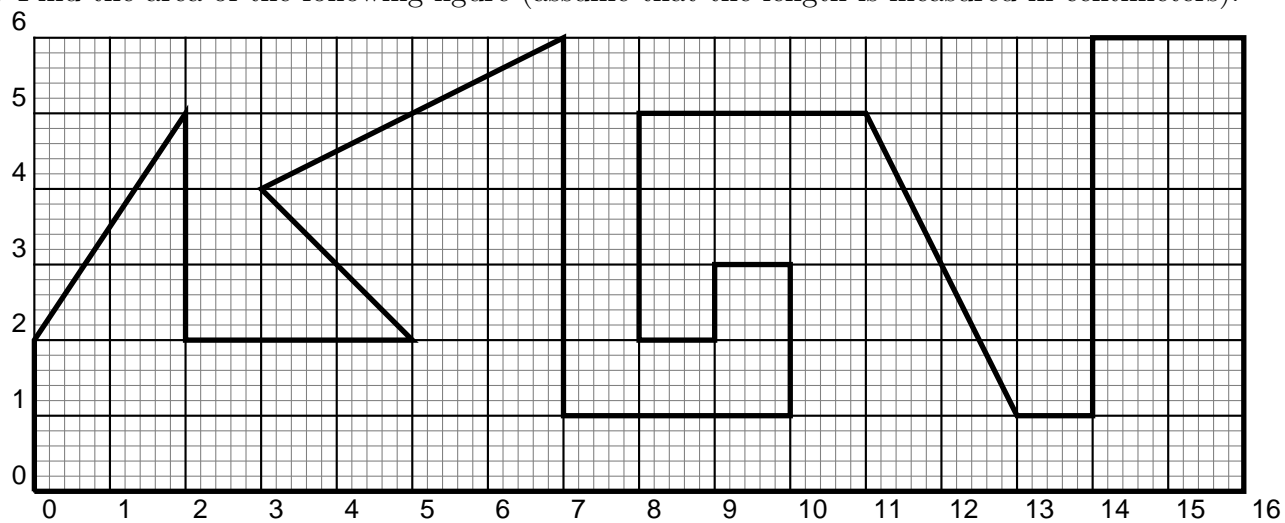
(g) $2\frac{1}{4} \div 5$

- (7) A page is $9\frac{3}{4}$ inches long. How many bands can be drawn on this page if each band is to be $\frac{3}{13}$ inches wide?

- (8) Find the area of the figure:



- (9) Find the area of the following figure (assume that the length is measured in centimeters):



8. COMBINED OPERATIONS AND COMPARING FRACTIONS

Recall the **order of operations** are:

- Simplify parentheses or any groupings (like complicated square roots) first;
- Powers and roots (exponents) next;
- Multiplications and divisions (left to right) next;
- Additions and subtractions (left to right) last.

(1) Simplify the following:

(a) $2\frac{1}{3} + 3\frac{1}{5} - \left(1\frac{2}{3}\right)^2$

(b) $\left(4 - 2\frac{3}{4}\right) \div \sqrt{\frac{4}{25}}$

(c) $2\frac{2}{3} \div 1\frac{2}{5} \cdot \left(2\frac{1}{3}\right)^2 - \left(\frac{1}{3} + \frac{2}{7}\right)$

(d) $3\frac{2}{5} \cdot 4\frac{2}{3} - 3^2 \div 2$

(e) $\sqrt{\frac{1}{25}} \cdot 2\frac{1}{5} - \frac{1}{3}$

(2) Arrange the following numbers in order, largest to smallest:

(a) $\frac{1}{3}, \frac{2}{13}, \frac{7}{39}$

(b) $2\frac{3}{5}, 2\frac{7}{15}, 2\frac{17}{30}$

(c) $3\frac{2}{7}, 3\frac{3}{8}, 3\frac{9}{28}$

(3) Use $<$, $>$, $=$ to fill in the blanks:

(a) $3\frac{1}{7}$ _____ $3\frac{3}{14}$

(b) $4\frac{8}{27}$ _____ $4\frac{11}{12}$

(c) $\frac{1}{18}$ of 20 _____ $\frac{3}{14}$ of 22

(4) The rate of interest on a credit card went from $8\frac{7}{10}$ % per annum to $8\frac{9}{13}$ % per annum. Did the rate of interest go up or down?

(6) What is the number of decimal places in a number? Explain using five examples.

(7) How do you round off a number to an indicated place? Explain using five examples.

(8) Round off each number to the indicated place:

(a) 8.123 to the tenth place

(b) 8.9076523 to the ten-thousandths place

(c) 7.907654 to four decimal places

(d) 123.7890674 to five decimal places

(e) 1234.98705 to the tens place

(f) 89765.34578098 to the hundreds place

(9) Find the sums:

$$23.78 + 901.234$$

$$0.009876 + 1.2345 + 78.09877$$

$$2.34509 + 234.78907 + 1.098765 + 0.0034526$$

$$34.90 + 12.876 + 23$$

(10) Find the differences:

$$1223.78 - 901.234$$

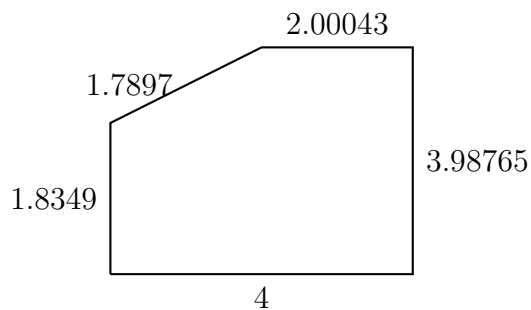
$$5.009876 - 1.2345$$

$$5432.34509 - 234.78907$$

$$34.90 - 12.876 + 23$$

- (11) Rob spent \$ 54.43 on a coat, \$ 64.23 on shoes, and \$ 38.48 on a shirt. How much did he spend in all?

- (12) Find the perimeter of the figure:



- (13) Subtract 32.478906 from 103

- (14) The temperature in Potsdam dipped from $32.9876^{\circ}F$ to $0.4561209^{\circ}F$. Find the difference in temperature.

10. MULTIPLICATION, DIVISION OF DECIMALS BY DECIMALS AND POWERS OF TEN

(1) Find the products:

$$0.003 \times 0.123$$

$$0.04 \times 0.0123$$

$$0.12 \times 0.0451$$

$$1.04 \times 2.098$$

$$12.3 \times 12.862$$

$$23.09 \times 13.43$$

(2) Find the quotients (do not round off):

$$25.2 \div 21$$

$$10.8 \div 5$$

$$2.072 \div 8$$

$$3.48 \div 16$$

$$4.41 \div 9$$

$$3.9765 \div 25$$

(3) Divide and round off the quotient to the indicated place:

$8.73 \div 6$

2 decimal places

$7.908 \div 12$

tenths place

$12.378 \div 11$

thousandths place

(4) Find the quotients (do not round off):

$26.65 \div 1.3$

$1.608 \div 1.2$

$1.6445 \div 0.11$

(5) Divide and round off the quotient to the indicated place:

$8.73 \div 1.1$

2 decimal places

$7.9095 \div 0.12$

tenths place

$12.378 \div 0.03$

thousandths place

(6) Perform the following operations:

8.73×100

7.908×10^5

12.378×10^2

$8.73 \div 10$

$7.908 \div 10^3$

$12.378 \div 100$

11. COMBINED OPERATIONS, CHANGING FRACTIONS TO DECIMALS, AND DECIMALS TO FRACTIONS, COMPARING DECIMALS

Recall the **order of operations** are:

- Simplify parentheses or any groupings (like complicated square roots) first;
- Powers and roots (exponents) next;
- Multiplications and divisions (left to right) next;
- Additions and subtractions (left to right) last.

(1) Find the value of the following:

(a) $3.5 + 7.2 \times 1.5 + 2.2$

(b) $12.05 + (1.2)^2 - 2.4 \times 0.6$

(c) $3.78 - 7.4 \div 3.7 + \sqrt{0.25}$

(d) $2.34 \times 10^3 - 45.7 \div 10^2 + 6.72 \div 1.2$

(e) $(2.3 + 4.1)^2 + 5$

(2) Change the following to decimals:

$$\frac{1}{4}$$

$$\frac{7}{12}$$

$$\frac{9}{40}$$

$$3\frac{3}{4}$$

$$8\frac{3}{20}$$

$$11\frac{7}{16}$$

(3) Change the following to fractions or mixed numbers and reduce the results to lowest terms:

$$0.2$$

$$1.23$$

$$1.75$$

$$2.84$$

$$3.85$$

$$5.55$$

(4) Arrange the decimals in order of size, with the largest first, using the symbol $>$.

(a) 0.520, 0.502, 0.250, 0.205, 2.05, 5.02

(b) 7, 7.01, 7.001, 7.0001, 7.00009

(c) 4.32, 4.302, 4.203, 4.322, 4.222

12. RATIO, RATE, AND PROPORTION PROBLEMS

(1) What is a ratio? Give five examples.

(2) Reduce the ratios to the lowest terms:

52 to 38

20 to 100

60 cents to a dollar

8 inches to a foot

$\frac{3}{5}$ to 4

4 to $\frac{2}{7}$

$3\frac{2}{3}$ to $\frac{2}{5}$

$4\frac{1}{3}$ to $1\frac{5}{7}$

1.34 to 0.67

(3) There are 18 female and 12 male students in a certain MTH 01 class. Find

- the ratio of the number of female students to the number of male students in the class.

- the ratio of the number of female students to the total number of students in the class.

- the ratio of the number of male students to the number of female students in the class.

(4) If the temperature in Potsdam is 7.8° and in Bronx is 19.5° , then find the ratio of the temperature in Potsdam to the temperature in Bronx.

(5) If a hall is $12\frac{3}{5}$ yards long and $9\frac{3}{7}$ yards wide, then find the ratio of the length of the room to its width.

(6) What is a proportion? Give five examples of proportions, and five examples which are not proportions.

(7) Are the following proportions?

$$\frac{8}{30} = \frac{12}{45}$$

$$\frac{\frac{3}{4}}{12} = \frac{1}{9}$$

$$\frac{25}{125} = \frac{0.5}{2}$$

(8) Solve for the letter in each proportion:

$$\frac{3}{16} = \frac{x}{8}$$

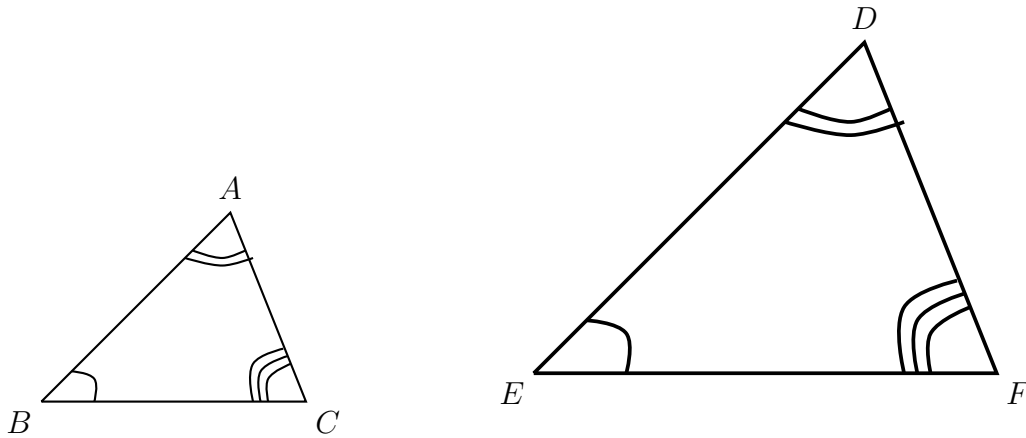
$$\frac{a}{12} = \frac{7}{48}$$

$$\frac{12}{25} = \frac{4}{b}$$

$$\frac{\left(\frac{1}{7}\right)}{x} = \frac{2}{\left(\frac{7}{12}\right)}$$

$$\frac{1.3}{7} = \frac{a}{0.35}$$

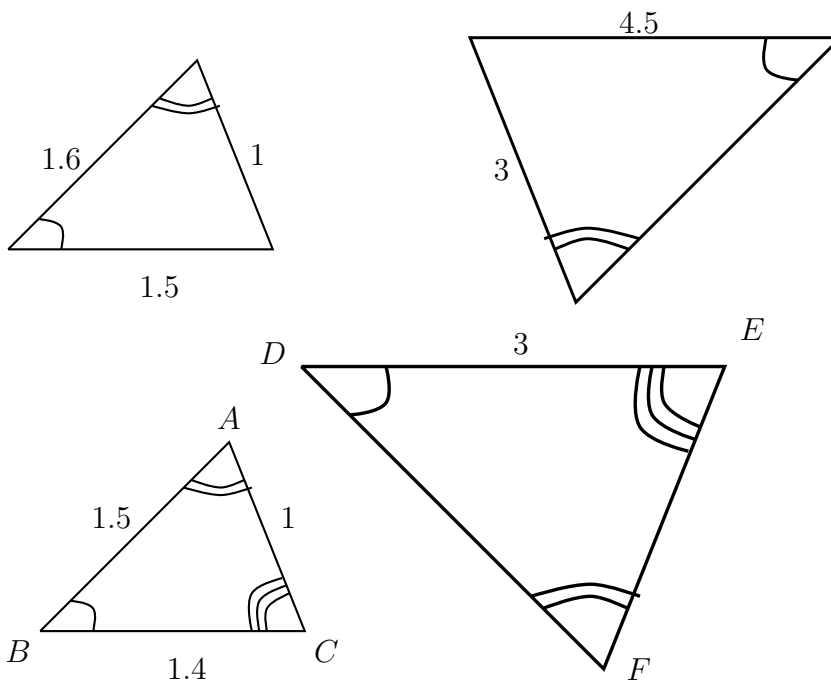
$$\frac{2\frac{1}{3}}{\frac{2}{5}} = \frac{b}{3\frac{4}{5}}$$

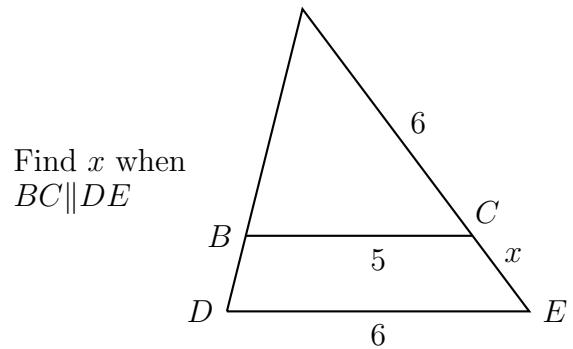
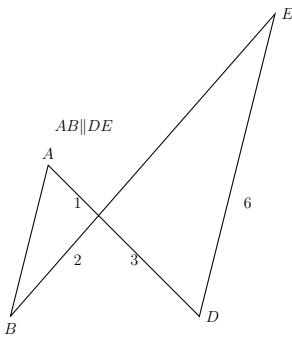
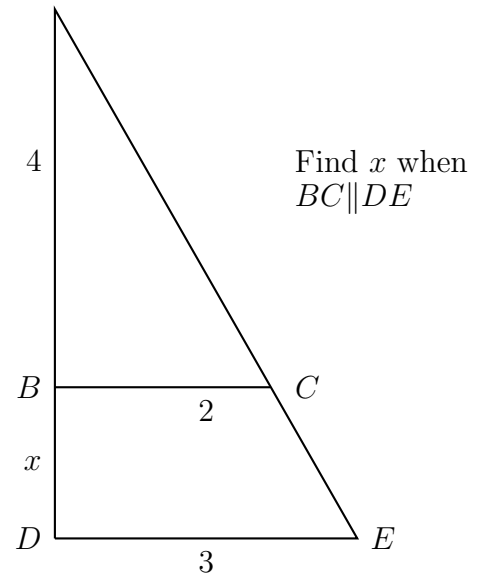
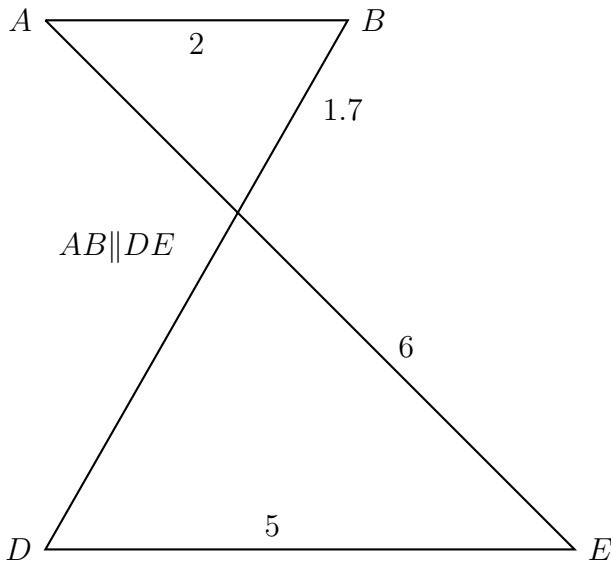
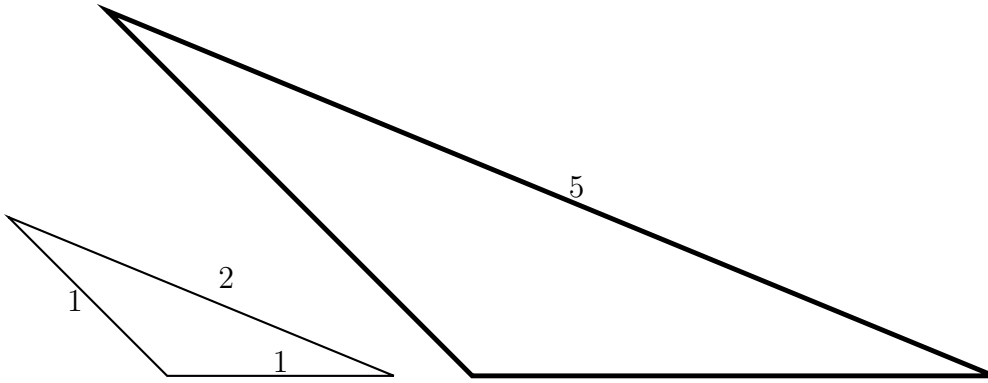
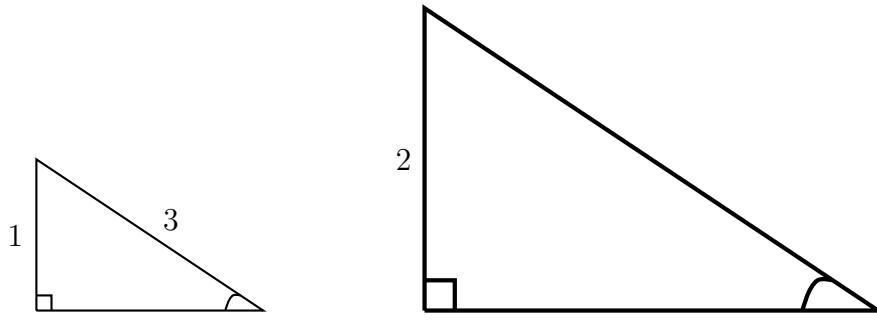


Definition. $\triangle ABC$ is **similar** to $\triangle DEF$, written $\triangle ABC \sim \triangle DEF$, if and only if, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, and $\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$. The ratio of the corresponding side lengths is called the **scale factor**. Two triangles are similar if they have the same shape.

Theorem. If two angles of one triangle are congruent, respectively, to two angles of a second triangle, then the two triangles are similar.

- (1) Assume that in the following figures, the triangles are similar. Find the lengths of the unknown sides.





14. PERCENTAGES

(1) What does **percent** mean? Explain using an example.

(2) Change each decimal to a percent:

(a) 0.5

(b) 0.245

(c) 3.479

(d) 7

(e) 0.0008

(3) Change each percent to a decimal. If the decimal is not exact, then round off to four decimal places.

(a) 34%

(b) 2%

(c) 0.34%

(d) $\frac{4}{5}\%$

(e) $3\frac{4}{15}\%$

(4) Change each fraction to a percent:

(a) $\frac{2}{5}$

(b) $\frac{3}{20}$

(c) $\frac{7}{12}$

(d) $\frac{4}{9}$

(e) $3\frac{2}{15}$

(5) Change each percent to a fraction:

(a) 35%

(b) 1%

(c) 3.5%

(d) 4,523%

(e) $\frac{3}{5}\%$

(f) $3\frac{5}{7}\%$

(6) Complete the table:

Fraction	Decimal	Percent
$\frac{3}{20}$		
	0.48	
		84 %
$2\frac{4}{25}$		
	5.408	
		$42\frac{2}{11}\%$

(7) Out of 80 students 35 students have textbooks. What percentage of the students have textbooks?

(8) In a building, out of 70 rooms 20 are heated. What percentage of the rooms are not heated?

15. FINDING A FRACTIONAL PART OF A NUMBER, PERCENT PROBLEMS AND WORD PROBLEMS
USING PERCENT

(1) Find

$$\frac{7}{8} \text{ of } 175$$

$$2\frac{4}{11} \text{ of } 143$$

$$3\frac{2}{19} \text{ of } 380$$

$$0.67 \text{ of } 75$$

$$2.11 \text{ of } 143$$

$$0.5 \text{ of } 380$$

$$12 \% \text{ of } 250$$

$$3\frac{2}{5} \% \text{ of } 250$$

$$0.75 \% \text{ of } 250$$

(2) Rob's weekly pay-checke is \$ 348.00. What is his weekly take-home pay if his total deductions amount to

- one-third his pay-checke;

- 34% his pay-checke;

- 0.38 his pay-checke.

(3) A certain used car has a sale price of \$ 6,750.00.

- You receive a discount of 25 % from the car dealer. How much would you need to pay to purchase that car?

- Suppose your mother offers to give you 25% of whatever amount you paid to purchase your car. How much did the car cost you in reality?

- If the car-dealer had offered you a 50% discount, and your mother had not offered to give you any money, then how much would you have paid for the same car?

(4) Answer the following:

(a) 12 is 20% of what number?

(b) 200 is 45% of what number?

(c) What percent of 18 is 9?

(d) What percent of 18 is 90?

(e) 13% of 85 is what number?

(f) 200 % of 78 is what number?

(5) Rob earns 11 % of the sales he makes. If he earns \$ 395.00 one week, then what was the gross amount of his sales? Write your final answer in the form of mixed number.

(6) A certain fruit contains 68 % of water by weight. How much water by weight does $4\frac{2}{9}$ pounds of the fruit contain? Write your final answer in the form of mixed number.

(7) A sweater that was originally priced at \$49.00 was being sold at \$ 35.00. What was the discount in percentage? Write your final answer in the form of mixed number.

16. SIGNED NUMBERS AND ADDING SIGNED NUMBERS

(1) What are the natural numbers?

(2) What are the whole numbers?

(3) What is a digit?

(4) What are the signed numbers?

(5) What is the sign of zero?

(6) What are the integers?

(7) What are the rational numbers?

(8) What are the irrational numbers?

(9) What are the real numbers?

(10) Can you suggest other types of numbers not mentioned above?

(11) How do we compare numbers?

(12) Write in words what the following mean:

(a) $<$

(b) $>$

(c) \leq

(d) \geq

(e) $x < 4$

(f) $-10 > -12$

(g) $0.9 < 1.1$

(h) $x \geq 3$

(13) State true or false:

(a) $3 < 10$

(b) $3 \leq 10$

(c) $-2 > -10$

(d) $10 \leq 10$

(e) $10 \geq 10$

(f) $1.2 > 1.5$

(g) $-1.34 < -2$

(h) $\frac{1}{7} < \frac{1}{9}$

(i) $-\frac{1}{7} < -\frac{1}{9}$

(j) $2\frac{1}{3} > 3\frac{1}{6}$

(k) $-2\frac{1}{3} > -3\frac{1}{6}$

(14) Fill in the blanks:

(a) The smallest natural number is _____.

(b) The largest natural number is _____.

(c) The smallest whole number is _____.

(d) The largest whole number is _____.

(e) The smallest integer is _____.

(f) The smallest positive integer is _____.

(g) The largest negative integer is _____.

(h) -3 _____ -10 .(i) 0 _____ -2 .(j) 10 _____ 10 .

(15) Plot the following numbers on the given number line:

 $0, -1, -2, -3, -4, 1, 2, 3, 4, 3.5, 0.75, -1.25, 1\frac{1}{4}, -2\frac{1}{3}, -3\frac{2}{5}$.

(16) Explain as though you are a teacher why $3 + 2 = 5$ (say, using 3 chocolates and 2 chocolates).

(17) Explain using the number line why $3 + 2 = 5$.

(18) Explain in your words why $10 + (-4) = 6$.

(19) Explain using the number line why $10 + (-4) = 6$.

(20) Explain using the number line what $-4 + (-5)$ is.

(21) Explain using the number line what $-7 + 6$ is. How about $6 + (-7)$?

(22) Write in your own words, how you would add two numbers of the same sign. Give five examples.

(23) Write in your own words, how you would add two numbers of different signs. Give five examples.

(24) What is the meaning of **absolute value**? Use the number line. Give five examples.

(25) What is the meaning of an **additive inverse**? How are additive inverses placed on the number line? Give five examples.

(26) Write in several ways as in the given example.

(a) $-2 - 3 = (-2) + (-3) = -2 - (+3) = -(+2) - (+3)$

(b) $12 + (-8) =$

(c) $-9 + 13 =$

(d) $7 + 8 =$

(e) $-5 + (-9) =$

(27) Find the sums:

(a) $12 + 13$

(b) $12 + (-13)$

(c) $-12 + (-13)$

(d) $-12 + 13$

(e) $(-34) + (9)$

(f) $(-34) + (-9)$

(g) $(-34) - (9)$

(h) $(34) + (-9)$

(i) $\left(-2\frac{1}{5}\right) + \left(3\frac{2}{3}\right)$

(j) $\left(-2\frac{1}{6}\right) + \left(-3\frac{2}{3}\right)$

(k) $\left(-5\frac{1}{8}\right) + \left(1\frac{3}{4}\right)$

(l) $-25.334 + 22.112$

(m) $1.392 + (-0.887)$

(n) $6.703 - 5.434$

(o) $25.67 - 98.10$

(p) $\left(-4\frac{1}{2}\right) + \left(-2\frac{5}{6}\right)$

(q) $\left(2\frac{1}{5}\right) + \left(-3\frac{2}{7}\right)$

(r) $\left(-3\frac{1}{12}\right) + \left(3\frac{2}{3}\right)$

(s) $\left(1\frac{1}{6}\right) + \left(3\frac{2}{3}\right)$

17. SUBTRACTING AND MULTIPLYING SIGNED NUMBERS

- (1) Here is a fun question. What do I mean when I say, “I am not not running.”?
- (2) Explain in your own words why $-(-3) = +3$. Maybe you could use the number line.
- (3) Find the differences:
- (a) $12 - (-8)$
- (b) $-12 - (-8)$
- (c) $-213 - (-52)$
- (d) $-3.567 - (-4.123)$
- (e) $-24.345 - (-43.123)$
- (f) $23.543 - (67.345)$
- (g) $\left(-3\frac{1}{2}\right) - \left(-5\frac{2}{7}\right)$
- (h) $\left(1\frac{1}{4}\right) - \left(-3\frac{2}{3}\right)$

(4) Answer the following:

(a) Subtract (-13) from 24.

(b) Subtract (-3.45) from -0.75 .

(c) Subtract $-3\frac{1}{4}$ from $-\frac{3}{8}$.

(d) Find the difference of -0.35 and -4.57 .

(e) Find the difference of $-1\frac{2}{3}$ and $2\frac{5}{6}$.

(f) The temperature in Potsdam rose from $-5^{\circ}F$ to $28^{\circ}F$. What was the rise in temperature?

(g) Water rushed from the depth of 3 feet below sea-level to a height of 12 feet above sea-level. What was the rise in water-level?

(5) Fill in the blanks:

• $+ \times + =$ _____

• $+ \times - =$ _____

• $- \times + =$ _____

• $- \times - =$ _____

(6) What is the meaning of 5 three times is 15?

(7) What is the meaning of -5 three times is -15 ? Do you now see $- \times + = -$?

(8) What is $(-1) \times 3$?

(9) What is $(-1) \times (-3)$?

(10) Multiply:

(a) $(-2)7$

(b) $(-2)(-7)$

(c) $(2)(-7)$

(d) $23(-8)$

(e) $(-11)(-9)$

(f) $(-11)(0)$

(g) $\left(-3\frac{1}{7}\right)\left(-2\frac{1}{3}\right)$

(h) $\left(-2\frac{1}{8}\right)\left(3\frac{1}{3}\right)$

(i) $\left(1\frac{2}{5}\right)\left(-2\frac{1}{7}\right)$

(j) $\left(-2\frac{3}{7}\right)\left(-5\frac{2}{3}\right)$

(11) What is a multiplicative inverse? Give 5 examples.

(12) Find the multiplicative inverse (or reciprocal) of:

(a) (-6)

(b) (2)

(c) $\frac{3}{4}$

(d) $-\frac{2}{3}$

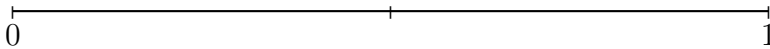
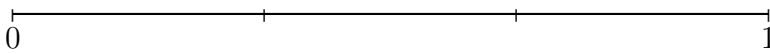
(e) $-3\frac{2}{3}$

(f) $-4\frac{1}{4}$

(g) $5\frac{2}{7}$

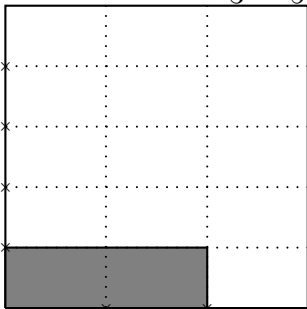
(h) $-5\frac{1}{8}$

(13) Using the two numberlines find the sum $\frac{1}{2} + \frac{2}{3}$.



(14) If the large square represents one, then what does the shaded region represent? Use this

picture to explain $\frac{2}{3} \times \frac{1}{5}$.



18. DIVIDING SIGNED NUMBERS, PROPERTIES, OPERATIONS WITH ZERO

- (1) $30 \div 5$ can be rewritten as $5\overline{)30}$. Here 30 is the **dividend** and 5 is the **divisor**. What is the **quotient** in this case?
- (2) Identify the divisor, dividend and quotient in each of the cases:
- (a) $48 \div 8 = 6$. Here, dividend = _____, divisor = _____, and quotient = _____.
- (b) $32 \div 4 = 8$. Here, dividend = _____, divisor = _____, and quotient = _____.
- (c) $a \div b = c$. Here, dividend = _____, divisor = _____, and quotient = _____.
- (3) Fill in the blanks:
- (a) $+\div+=$ _____
- (b) $+\div- =$ _____
- (c) $-\div+=$ _____
- (d) $-\div- =$ _____
- (4) What is the meaning of 15 divided by three is 5?
- (5) What is the meaning of -15 divided by three is -5 ? Do you now see $-\div+=-$?
- (6) What is $(-3) \div 1$?
- (7) What is $(-3) \div (-1)$?
- (8) Divide:
- (a) $14 \div (-2)$
- (b) $(-14) \div (-7)$
- (c) $(-14) \div (-2)$
- (d) $24 \div (-8)$
- (e) $(-44) \div (-9)$
- (f) $(-11) \div (0)$
- (g) $(0) \div (-11)$

(9) Answer the following:

- How many threes make fifteen?
- How many 3's make 15?
- What is $15 \div 3$?

(10) Answer the following:

- How many quarters make three-quarters?
- How many $\left(\frac{1}{4}\right)$'s make $\left(\frac{3}{4}\right)$?
- What is $\left(\frac{3}{4}\right) \div \left(\frac{1}{4}\right)$?
- What is $\left(\frac{3}{4}\right) \times \left(\frac{4}{1}\right)$?

(11) Answer the following:

- How many two-thirds make eight-thirds?
- How many $\left(\frac{2}{3}\right)$'s make $\left(\frac{8}{3}\right)$?
- What is $\left(\frac{8}{3}\right) \div \left(\frac{2}{3}\right)$?
- What is $\left(\frac{8}{3}\right) \times \left(\frac{3}{2}\right)$?

(12) How do you divide a fraction by another fraction? Do you need common denominators in this case?

(13) Divide:

(a) $\frac{3}{4} \div \frac{1}{3}$

(b) $\left(-\frac{4}{5}\right) \div \left(\frac{3}{4}\right)$

(c) $\left(-\frac{11}{13}\right) \div \left(-\frac{26}{33}\right)$

(d) $\left(-3\frac{1}{7}\right) \div \left(-2\frac{1}{3}\right)$

(e) $\left(-4\frac{3}{5}\right) \div \left(5\frac{2}{7}\right)$

(14) State the commutative property of addition. Give 5 examples.

(15) State the commutative property of multiplication. Give 5 examples.

(16) State the associative property of addition. Give 5 examples.

(17) State the associative property of multiplication. Give 5 examples.

(18) Does the commutative property hold for subtraction? Explain using an example.

(19) Does the commutative property hold for division? Explain using an example.

(20) Does the associative property hold for subtraction? Explain using an example.

(21) Does the associative property hold for division? Explain using an example.

(22) State true or false. If true, then give reason why. If false, then give an example explaining why.

(a) $2 + 4 = 4 + 2$

(b) $a + 8 = 8 + a$

(c) $a - 8 = 8 - a$

(d) $a8 = 8a$

(e) $a \div 8 = 8 \div a$

(f) $a - (8 - 3) = (a - 8) - 3$

(g) $a \times (8 \times 3) = (a \times 8) \times 3$

(h) $a \div (8 \div 3) = (a \div 8) \div 3$.

(i) $(-5)(-4) = (-4)(-5)$

(j) $(-5) \div (-4) = (-4) \div (-5)$

(k) $(-5) - (-4) = (-4) - (-5)$

(l) $(-5) + (-4) = (-4) + (-5)$

(23) What is the additive identity property of 0? Give 5 examples.

(24) Give 5 examples of subtractions involving 0.

(25) Give 5 examples of multiplications involving 0.

(26) Give 5 examples of divisions involving 0. What is it that we should be careful about in this situation?

(27) Find the value if possible.

(a) 2×0

(b) $3 - 0$

(c) 0×3

(d) $0 - 2$

(e) $2 \div 0$

(f) $0 \div 2$

(g) $0 \div 0$

(h) $\frac{2 - 3}{0 - 3}$

(i) $0 - (-2)$

(j) $\frac{5 + (-5)}{-2 + 2}$

$$(k) \frac{12 + (-5)}{-8 + 2}$$

$$(l) \frac{\left(\frac{2}{3}\right)}{12}$$

$$(m) \frac{12}{\left(\frac{2}{3}\right)}$$

$$(n) \frac{\left(\frac{2}{3} - \frac{1}{5}\right)}{\left(\frac{1}{3} + \frac{2}{5}\right)}$$

$$(o) \frac{\left(\frac{2}{3} \times \frac{1}{5}\right)}{\left(\frac{1}{3} \div \frac{2}{5}\right)}$$

$$(p) \frac{\left(\frac{2}{3} - \frac{1}{5} + \frac{1}{2}\right)}{\left(\frac{1}{3} + \frac{2}{5} - \frac{1}{2}\right)}$$

$$(q) \frac{\left(\frac{1}{3} - \frac{3}{5} + \frac{1}{2}\right)}{3}$$

$$(r) \frac{4}{\left(\frac{1}{3} - \frac{3}{5} + \frac{1}{2}\right)}$$

19. POWERS AND SQUARE ROOTS OF SIGNED NUMBERS

(1) In the expression 2^5 , the number 2 is the **base** and the number 5 is the **exponent**. Identify the base and the exponent in the following expressions:

(a) 3^4 . Here the base is _____ and the exponent is _____.

(b) $(-3)^4$. Here the base is _____ and the exponent is _____.

(c) -3^4 . Here the base is _____ and the exponent is _____.

(d) $\left(\frac{2}{3}\right)^3$. Here the base is _____ and the exponent is _____.

(2) Evaluate the following:

(a) 2^5

(b) $(-2)^6$

(c) -2^6

(d) 2^0

(e) 0^0

(f) 0^2

(g) $-(-4^2)$

(h) $-(-4)^2$

(i) $\left(\frac{3}{4}\right)^3$

(j) $\left(\frac{-3}{4}\right)^3$

(k) $\left(\frac{3}{-4}\right)^3$

(3) Complete the following table of squares (we had done this before):

$0^2 =$	$6^2 =$	$12^2 =$	$18^2 =$
$1^2 =$	$7^2 =$	$13^2 =$	$19^2 =$
$2^2 =$	$8^2 =$	$14^2 =$	$20^2 =$
$3^2 =$	$9^2 =$	$15^2 =$	$30^2 =$
$4^2 =$	$10^2 =$	$16^2 =$	$40^2 =$
$5^2 =$	$11^2 =$	$17^2 =$	$50^2 =$

(4) Complete the following table of cubes (we had done this before):

$0^3 =$	$3^3 =$	$6^3 =$	$9^3 =$
$1^3 =$	$4^3 =$	$7^3 =$	$10^3 =$
$2^3 =$	$5^3 =$	$8^3 =$	$100^3 =$

(5) What is the principal square root?

(6) What is the radicand?

(7) What is a rational number? Give 5 examples.

(8) What is an irrational number? Give 5 examples.

(9) What are real numbers? Give 5 examples.

(10) Give 5 examples of imaginary numbers.

(11) Is the number $0.777\cdots$ rational? Explain.

Solution. Yes, the number $0.777\cdots$ is rational. Let $x = 0.777\cdots$. Then $10x = \underline{\hspace{2cm}}$.

Hence, $10x - x = \underline{\hspace{2cm}} - 0.777\cdots$.

So, $9x = \underline{\hspace{2cm}}$.

Thus, $x = \underline{\hspace{2cm}}$, which is a quotient of two integers. Therefore x is a rational number.

(12) Is the number $0.767676\cdots$ rational? Explain.

(13) Identify the radicand in each expression:

(a) $\sqrt{18}$

(b) $\sqrt{x^2 + 1}$

(c) $\sqrt{\frac{x}{y}}$

(14) Complete the following table of square roots:

$\sqrt{0} =$	$= 6$	$= 12$	$= 18$
$\sqrt{1} =$	$= 7$	$= 13$	$= 19$
$\sqrt{4} =$	$= 8$	$= 14$	$= 20$
$\sqrt{9} =$	$= 9$	$= 15$	$= 30$
$= 4$	$= 10$	$= 16$	$= 40$
$= 5$	$= 11$	$= 17$	$= 50$

(15) Complete the following table of cube roots:

$\sqrt[3]{0} =$	$= 3$	$= 6$	$= 9$
$\sqrt[3]{1} =$	$= 4$	$= 7$	$= 10$
$\sqrt[3]{8} =$	$= 5$	$= 8$	$= 100$

Write down in your notebook the following **important** properties of square-roots: ($a \geq 0, b > 0$)

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$(\sqrt{a})^2 = a$$

Answer the following:

(1) Is $\sqrt{a^2 + b^2} = a + b$ for every a, b ? Explain.

(2) Is $\sqrt{a^2 - b^2} = a - b$ for every a, b ? Explain.

(3) Is $\sqrt{\frac{1}{a^2 + b^2}} = \frac{1}{a + b}$ for every a, b ? Explain.

(4) Is $\sqrt{\frac{1}{a^2 + b^2}} = \frac{1}{a} + \frac{1}{b}$ for every a, b ? Explain.

(5) Find or simplify each square root:

(a) $\sqrt{49}$

(b) $\sqrt{81}$

(c) $\sqrt{169}$

(d) $-\sqrt{\frac{49}{81}}$

(e) $\sqrt{45}$

(f) $\sqrt{72}$

(g) $\sqrt{192}$

(h) $\sqrt{121 + 100}$

(i) $\sqrt{121} + \sqrt{100}$

(j) $\sqrt{121 - 100}$

(k) $\sqrt{121} - \sqrt{100}$

(l) $(\sqrt{5})^2$

(m) $\sqrt{8} \cdot \sqrt{2}$

$$(n) \sqrt{\frac{169}{9}}$$

$$(o) -\sqrt{\frac{900}{289}}$$

$$(p) \frac{3}{\sqrt{6}}$$

$$(q) \frac{5}{\sqrt{5}}$$

$$(r) \frac{7}{\sqrt{17}}$$

$$(s) \frac{1}{\sqrt{11}}$$

$$(t) \sqrt{75}$$

$$(u) \sqrt{200}$$

$$(v) \sqrt{1000}$$

$$(w) \sqrt{\frac{324}{25}}$$

$$(x) \sqrt{\frac{1600}{49}}$$

20. SCIENTIFIC NOTATION

(1) Write the following in scientific notation:

- 3,209,000,000
- 90,120,210,320
- $-123,200,000,000$
- 0.00000000234
- -0.00024
- 3

(2) Multiply and write the product in scientific notation:

- $(3 \times 10^7) \times (2 \times 10^2)$
- $(-2 \times 10^4) \times (4 \times 10^9)$
- $(4 \times 10^5) \times (5 \times 10^7)$
- $(7 \times 10^3) \times (-4 \times 10^2)$
- $(2 \times 10^{-7}) \times (2 \times 10^2)$
- $(-3 \times 10^4) \times (3 \times 10^{-9})$
- $(4 \times 10^{-5}) \times (3 \times 10^{-7})$
- $(-6 \times 10^3) \times (-4 \times 10^{-2})$
- $(4.1 \times 10^5) \times (5.2 \times 10^7)$
- $(7.4 \times 10^3) \times (-4.3 \times 10^2)$
- $(2.2 \times 10^{-7}) \times (2.3 \times 10^2)$
- $(-3.3 \times 10^4) \times (3.1 \times 10^{-9})$

(3) Divide and write the quotient in scientific notation:

- $(3 \times 10^7) \div (2 \times 10^2)$
- $(-2 \times 10^4) \div (4 \times 10^9)$
- $(4 \times 10^5) \div (5 \times 10^7)$
- $(7 \times 10^3) \div (-4 \times 10^2)$

- $(2 \times 10^{-7}) \div (2 \times 10^2)$

- $(-3 \times 10^4) \div (3 \times 10^{-9})$

- $(4 \times 10^{-5}) \div (3 \times 10^{-7})$

- $(-6 \times 10^3) \div (-4 \times 10^{-2})$

- $(7.2 \times 10^5) \div (1.2 \times 10^7)$

- $(7.8 \times 10^3) \div (-1.3 \times 10^2)$

- $(1.26 \times 10^{-7}) \div (2.1 \times 10^2)$

- $(-6.3 \times 10^4) \div (7 \times 10^{-9})$

21. ORDER OF OPERATIONS, AND GROUPING SYMBOLS

(1) Write down the **order of operations**.

(2) Evaluate:

(a) $2 - 3 - 4$

(b) $2 - (3 - 4)$

(c) $12 \div 4 \div 2$

(d) $12 \div (4 \div 2)$

(e) $(-2)^3 - \sqrt{225} + 81 \div 3 + 6$

(f) $3(12 - 4)^2 - \sqrt{48 \div 3}$

(g) $\left(-\frac{3}{2}\right)^3 + \left(3\frac{1}{2}\right)\left(\frac{1}{2}\right)^2$

(h) $\left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$

(i) $\left(\frac{3}{2} - \frac{1}{2}\right)^2$

(j) $\left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

Evaluate

$$(1) -4[12 - 2(-7)]$$

$$(2) 2^3 - 5\{6^2 - \sqrt{3^2 + 4^2} - (-(-20))\}$$

$$(3) \frac{(-2)^2 + 5}{3(12 - 7)}$$

$$(4) (3 - 7 - 8) - \sqrt{36 + 64}$$

$$(5) 2(3 + 2)^2 - [4 - (2 + 7\{3 - 9\})]$$

$$(6) \frac{7^2 - 5^2}{7 - 5}$$

$$(7) \frac{3^2 - 4^2}{11} + \frac{(-3)^2 + 4^2}{11}$$

$$(8) \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)^2 + 3\left(\frac{1}{2}\right)^2\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^3$$

$$(9) \left(\frac{1}{2}\right)^3 - \left(\frac{1}{3}\right)^3$$

$$(10) \left(\frac{1}{2} - \frac{1}{3}\right)^3$$

22. EVALUATING EXPRESSIONS AND FORMULAS

Evaluate the expressions for the given values of variables.

(1) $x^2 - 3x + 4$

(a) $x = 0$

(b) $x = 2$

(c) $x = -2$

(2) $\left(\frac{2a - b}{3a - b^2 + 4} \right)$

(a) $a = 0, b = 0$

(b) $a = -2, b = 3$

(c) $a = -2, b = -3$

(3) $\sqrt{\frac{x^2 + y^2}{x^2 y^2}}$

(a) $x = 1, y = -1$

(b) $x = -2, y = 3$

(c) $x = -2, y = -3$

$$(4) 2[-x - y]^2 + 3(y - x)$$

$$(a) x = 0, y = -3$$

$$(b) x = -1, y = -2$$

$$(c) x = 4, y = -1$$

$$(5) \frac{3x + y}{4xy}$$

$$(a) x = 0, y = -3$$

$$(b) x = -1, y = -2$$

$$(c) x = 4, y = -1$$

$$(6) a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a) a = 2, b = 3$$

$$(b) a = 0, b = -2$$

$$(c) a = -1, b = -3$$

$$(7) (a - b)^3$$

$$(a) a = 2, b = 3$$

$$(b) a = 0, b = -2$$

$$(c) a = -1, b = -3$$

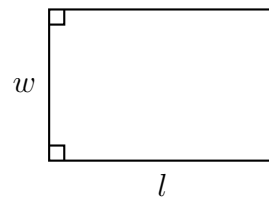
Evaluate the formulae for the given values of variables.

(1) $A = lw$

(a) $l = 3, w = 4$

(b) $l = 12, w = 22$

(c) $l = 30, w = 20$

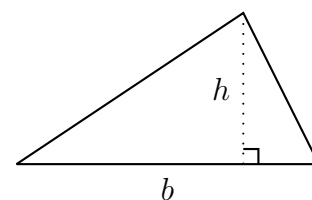


(2) $A = \frac{bh}{2}$

(a) $b = 3, h = 10$

(b) $b = 22, h = 13$

(c) $b = 7, h = 9$

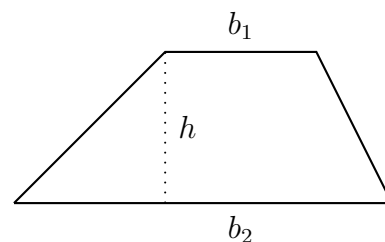


(3) $A = \frac{(b_1 + b_2)h}{2}$

(a) $b_1 = 2, b_2 = 3, h = 5$

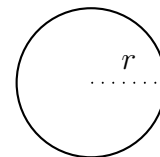
(b) $b_1 = 4, b_2 = 12, h = 10$

(c) $b_1 = 12, b_2 = 14, h = 8$



(4) $A = \pi r^2$ (Don't let π trouble you. Leave it in your answer.)

(a) $r = 3$

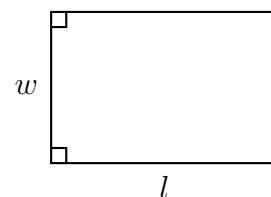


(b) $r = \frac{3}{2}$

(c) $r = \sqrt{5}$

(5) $P = 2l + 2w$

(a) $l = 3, w = 23$

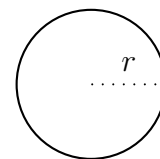


(b) $l = 14, w = 15$

(c) $l = 22, w = 29$

(6) $C = 2\pi r$

(a) $r = 3$

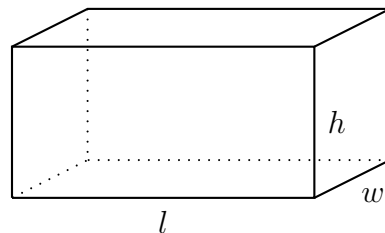


(b) $r = \frac{3}{2}$

(c) $r = \sqrt{5}$

$$(7) V = lwh$$

$$(a) l = 2, w = 3, h = 5$$

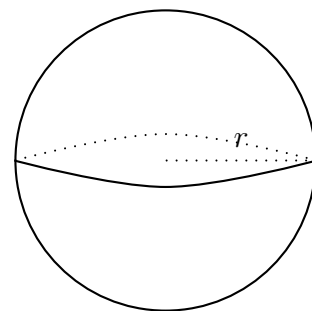


$$(b) l = 4, w = 12, h = 10$$

$$(c) l = 12, w = 14, h = 8$$

$$(8) V = \frac{4}{3}\pi r^3$$

$$(a) r = 3$$



$$(b) r = \frac{3}{2}$$

$$(c) r = \sqrt{5}$$

$$(9) m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(a) x_1 = -1, x_2 = -3, y_1 = 5, y_2 = 4$$

$$(b) x_1 = -100, x_2 = -100, y_1 = 5, y_2 = 4$$

$$(c) x_1 = -1, x_2 = -3, y_1 = 4, y_2 = 4$$

23. SOLVING LINEAR EQUATIONS

Here we work with equations. An **equation** is a mathematical statement involving equality ($=$). A solution of an equation is a number that when substituted for the variable gives a true statement.

For example, $x = 3$ **solves** the equations $2x = 6$ and $x - 5 = -2$.

(1) Is -4 a solution of $2x = 6$?

(2) Is -3 a solution of $x = 3$?

(3) Is 1 a solution of $5x = 0$?

(4) Is -5 a solution of $-3x = -15$?

(5) Write 5 equations which are solved by -4 .

(6) Write 5 equations which are solved by -5 .

(7) Write 5 equations which are solved by 2 .

Think of an $=$ sign as a **balance**. Whatever you do to one side, you must do to the other side as well. So,

- The **same** number may be **added** to both sides.
- The **same** number may be **subtracted** from both sides.
- Both sides may be **multiplied** by the **same** number.
- Both sides may be **divided** by the **same** non-zero number.

Solve the equations.

$$(1) x + 3 = 11$$

$$(2) 12 = x - 4$$

$$(3) -18 + x = 20$$

$$(4) 2x = 10$$

$$(5) -20 = \frac{x}{5}$$

$$(6) -30 = -6x$$

$$(7) \frac{x}{2} = 10$$

$$(8) -x + 4 = 7$$

$$(9) 2x + 3 = 11$$

$$(10) 12 = 3x - 4$$

$$(11) -18 + 4x = 20$$

$$(12) 2x - 12 = 10$$

$$(13) -x + \frac{1}{3} = 4$$

$$(14) -20 = \frac{x}{5} + \frac{4}{3}$$

$$(15) -30 = -6x + 7$$

$$(16) \frac{x}{2} - \frac{1}{3} = 12$$

$$(17) \frac{x-1}{12} = 2$$

$$(18) -\frac{3x}{2} - \frac{1}{3} = 8$$

$$(19) -\frac{5x}{2} - \frac{11}{3} = \frac{3}{5}$$

24. PRACTICE PROBLEMS

(1) The **absolute value** of a number is its distance from zero. Evaluate:

(a) $|5|$

(b) $|-3|$

(c) $|0|$

(d) $|-6|$

(e) $|12|$

(f) $|23|$

(g) $|-123|$

(2) The **opposite** or the **additive inverse** of a number is the number with opposite sign. The opposite of a is $-a$. The **Additive Inverse property** states that $a + (-a) = 0$.

(a) The opposite of $+3$ is -3 . Therefore, $(+3) + (-3) = 0$.

(b) The opposite of $+5$ is _____. Therefore, $(+5) + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

(c) The opposite of -7 is _____. Therefore, $(-7) + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

(d) The opposite of $\frac{2}{3}$ is _____. Therefore, $\left(\frac{2}{3}\right) + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

(e) The opposite of $-\frac{1}{5}$ is _____. Therefore, $\left(-\frac{1}{5}\right) + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

(3) Find the sums:

(a) $6 + 3$

(b) $(5) + (2)$

(c) $(-4) + (-3)$

(d) $(-7) + (-2)$

(e) $(-10) + (-11)$

(f) $(-14) + (-2)$

(g) $(-4) + (+1)$

(h) $(+7) + (-10)$

(i) $8 + (-2)$

(j) $-15 + 27$

(k) $(-25) + (-13)$

(l) $-32 + 55$

(m) $-32 + (-44)$

(4) Find the differences:

(a) $6 - 3$

(b) $(5) - (2)$

(c) $(-4) - (-3)$

(d) $(-7) - (-2)$

(e) $(-10) - (-11)$

(f) $(-14) - (-2)$

(g) $(-4) - (+1)$

(h) $(+7) - (-10)$

(i) $8 - (-2)$

(j) $-15 - 27$

(k) $(-25) - (-13)$

(l) $-32 - 55$

(m) $-32 - (-44)$

(5) Evaluate:

(a) $(-8) - (-2)$

(b) $(-9) + (-2)$

(c) $9 - 16$

(d) $-6 + 9$

(e) $-7 - (-3)$

(f) $(-6) - 0$

(g) $-5 - 3 - 2$

(h) $-\sqrt{25}$

(i) $-2\sqrt{25}$

(j) $(-3)^2$

(k) -3^2

(l) $-(-3)^3$

(m) 10^6

(n) $-7 - 2 + 5$

(o) $-8 + 3 - 5 + 4 - 7$

(p) $(-8)(+3)(-5)(+4)(-7)$

(6) Evaluate:

(a) $7106 \div 34$

(b) $1517 \div 37$

(c) $7 + 3(5 + 6)$

(d) $8 - 3[4 - (5 - 7)]$

(e) $\frac{3}{5}$ of 25

(f) $\frac{3}{4}$ of 24

(g) $\frac{1}{9} + \frac{5}{6}$

(h) $\frac{2}{15} + \frac{5}{12}$

(i) $8\frac{1}{3} - 5\frac{3}{4}$

(j) $12\frac{2}{5} - 9\frac{1}{3}$

(k) $5 \div \frac{2}{3}$

(l) $3\frac{2}{5} \div 1\frac{3}{7}$

(m) $34.768 + 59 + 7.12$

(n) $4.78 + 3.897 + 8$

(o) $56.8 - 5.36$

(p) $78.3 - 9.34$

(q) 0.89×0.007

(r) 0.65×0.003

(s) $6.75 \div 0.8$

(t) $4.94 \div 0.4$

(u) $-3 - 5 + 9 - (-2)$

(v) $-20 \div 4 \times 7$

(w) $4(-3) + 15 \div 5$

(x) $-2(-7 + 5)$

(y) $2 \cdot 5^2 - 9$

(7) Evaluate:

(a) $-7^2 + (-5)^2$

(b) $5 + 3(6 - 2 \cdot 5)$

(c) $-3^2 + [5 + (-1)]^2$

(d) 30% of 120.

(e) 60 % of 50.

(f) $\frac{7}{8}$ of 32.

(g) $\frac{2}{3}$ of 30.

(h) 0.35 of 36.

(i) 0.10 of 63.50.

(j) $6\frac{2}{3}\%$ of 504.

(k) $\frac{1}{4}\%$ of 200.

(l) $8\frac{1}{4}\%$ of 63.50.

(8) Change each percent to decimal:

(a) 50%

(b) 7%

(c) 6.5%

(d) $8\frac{3}{4}\%$

(9) Change each percent to a fraction in lowest terms:

(a) 50%

(b) 7%

(c) 6.5%

(d) $8\frac{3}{4}\%$

(10) Answer the following:

(a) Convert $\frac{3}{8}$ to a decimal rounded to the nearest hundredth.

(b) Convert $\frac{4}{7}$ to a decimal rounded to the nearest thousandth.

(c) Change each of the following fractions into mixed numbers:

- $\frac{7}{4}$
- $\frac{11}{3}$
- $\frac{56}{15}$

(d) Change each of the following mixed numbers into improper fractions:

- $2\frac{3}{5}$
- $4\frac{1}{2}$
- $15\frac{3}{4}$

(e) Reduce each fraction to its lowest terms:

- $\frac{6}{9}$
- $\frac{42}{42}$
- $\frac{84}{33}$
- $\frac{57}{48}$
- $\frac{64}{6}$
- $\frac{6}{16}$

(f) What is the average of 86, 75, 80, 78, 84, and 89?

(g) What is the average of 70, 80, 90, 60, and 85?

(h) Arrange the following in order from largest to smallest:

- $\frac{5}{7}, \frac{7}{12}, \frac{3}{4}$
- $\frac{9}{4}, \frac{3}{1}, \frac{6}{5}$
- $\frac{9}{9}, \frac{3}{3}, \frac{12}{5}$
- $\frac{14}{14}, \frac{4}{4}, \frac{7}{7}$

(i) Arrange the following in order from largest to smallest: 2.05, 2.502, 2.5, 2.123, 0.29, 20.1.

(j) What is the correct numeral for forty-two million, three hundred forty thousand and twelve?

(k) Write out the words for the number 3, 582, 638.

(l) 16 is 20% of what number?

(m) 90 is what percent of 225?

(n) 32% of what number is 256?

(o) What percent of 6 is 12?

(p) Write the correct symbol $<$, $>$, $=$:

- 6 1
- 0 - 5
- -2 - 7
- -1 3

(q) Multiply each of the following numbers by 100:

- 45
- 2.3
- 4.56
- 0.67

- 0.532

(r) Divide each of the following numbers by 100, and write result as a decimal number:

- 25
- 2
- 8.5
- $6\frac{12}{25}$

(s) For each of the following, find the LCD and rename each fraction:

- $\frac{1}{2}, \frac{3}{4}$
- $\frac{2}{5}, \frac{1}{3}$
- $\frac{7}{12}, \frac{4}{15}$
- $\frac{2}{9}, \frac{7}{15}, \frac{3}{20}$

(t) Assume the following sets are denominators of fractions. Find the LCD:

- {4, 5, 10}
- {3, 6, 9}
- {6, 12}
- {4, 9}
- {4, 15, 18}
- {6, 13, 26}

(11) Solve :

(a) $\frac{x}{5} = \frac{12}{15}$

(b) $x - 8 = 11$

(c) $3x + 7 = 13$

(d) $5x = -15$

(e) $\frac{x}{-2} = -4$

(f) $15 \left(x - \frac{3}{5} \right) = 15 \left(\frac{1}{3} \right)$

(12) Word problems:

- (a) A rectangular field is 12 feet long and 10 feet wide. How many feet of fencing material are needed to enclose the entire field?
- (b) A rectangular room is 15 feet long and 12 feet wide. How is the area of the floor?
- (c) What is the cost to replace a 4 ft. by 5 ft. window, if the cost of glass is \$ 2.45 per square foot?

- (d) If carpet sells for \$ 14 per square yard, then how much would it cost to carpet a room that is 4 yards by 7 yards?
- (e) Suppose subway tokens cost \$ 1.50. How much change should get get from a \$ 20 bill, if you buy 8 tokens?
- (f) A washing machine costs \$ 250 plus 80% tax. What is the total cost?
- (g) George buys twelve gallons of gas at \$ 2.49 per gallon. If he pays a \$ 50, how much would he get back?
- (h) A stereo costs \$ 99 plus $8\frac{1}{4}\%$ tax. What is the total cost?
- (i) The regular cost of a phone call is \$ 7.70. The same call made after 11 PM is reduced by 60%. What is the reduced price?
- (j) A CD costs \$ 12.95. The price is increased 20%. What is the new price?
- (k) A 6-ft. man casts a 5-ft. shadow. Find the height of a tree that casts a 40-ft. shadow.
- (l) On a mathematics test of 40 questions, Yoshi solved 85% of the problems correctly. How many did he answer correctly? How many did he answer incorrectly?
- (m) In purchasing a house, a 15% down payment is required. Find the amount of down payment on a house that sells for \$ 140,000. Find the amount left to pay.
- (n) A car listed at \$ 23,500 was sold at a 20% discount. What was the selling price?
- (o) A woman used 10 gallons of gas on a 180 mile trip. How many gallons of gas can she expect to use on a 300 mile trip?
- (p) Seven builders finish 10 houses in a month. How many houses could 35 builders finish in a month?
- (q) A store has a bargain price of 85¢ for three jars of grape jelly. How many jars could one buy for \$ 5.94?
- (r) On a map, half an inch represents 12 miles. How many miles would two and a quarter inches represent?