Final exam review guide. Calculus I - MATH1101, Section 005. Spring 2023.

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This is a list of topics that you should know well from each section, and which exercises from the book you can do practice that topic.

NOTE: The list of exercises is very long. It does not mean that you have to do all these exercises. Rather, for each topic in each section, try a couple of exercises. If everything is very clear, move on to the next topic; otherwise try a few more exercises and ask for help if you need.

The topics that I expect you to know very well (and those that will probably be in the final) are marked with a * ∗

- **Section 2.2**
  - * Find limits given the graph of a function. Ex. 4–9.
  - * Understand and find infinite limits. Ex. 29–40.
- **Section 2.3**
  - * Find limits using the limit laws. Ex. 11-34. NOTE: you can use L'Hopital's rule here if convenient.
  - Understanding the limit laws. Ex. 61, 62.
  - Squeeze theorem. Ex. 39–42.
- **Section 2.5**
  - Memorize: definition of continuity.
  - * Understand continuity given the graph of a function. Ex. 3–6. Ex. 7–10.
  - * Understand continuity given expression of a function. Ex. 21–24, Ex. 47–50, Ex. 51(a),(b).
  - Memorize and understand the Intermediate Value Theorem. Ex. 55–58.
- **Section 2.6**
  - * Understand limits as $x \to \pm\infty$ given the graph. Ex. 3–9.
  - * Evaluate limits as $x \to \pm\infty$. Ex. 15–36. NOTE: you can use L'Hopital's rule here if you need it.
  - * Finding vertical and horizontal asymptotes. Ex. 47–51.
- **Section 2.7**
  - * Memorize: definition of derivative: $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$.
  - Find the derivative of a function using the definition. Ex. 21–24.
  - * Understand the concept of derivative in three ways: as the slope of the tangent to the graph, as the instantaneous rate of change, and as velocity. Ex. 15–17, Ex. 50–51.
- **Section 2.8**
  - * Memorize: definition of derivative (again; it is the same as in sec. 2.7 with x instead of a).
  - * Given the graph of a function, sketch the graph of its derivative. Ex. 1–3, Ex. 4–11. Ex. 47–51.
  - Find the derivative of a function using the definition. Ex. 21–28,
  - * Understand where a function is *not* differentiable given the graph. Ex. 41–44.

For Chapter 3, **it is important that you know the differentiation rules well and can take the derivative of any function correctly and quickly**. For this, you only need practice. I list a lot of exercises in case you need to practice this skill, but if you know it well already, you can skip most of the exercises listed in 3.1–3.4.

- **Section 3.1**
  - * Find derivatives of polynomials and exponentials. Ex. 3–34.
  - * Find the equation of the tangent line at a point of a graph. Ex. 37–44. Ex. 59–61.
- **Section 3.2**
  - * Find derivatives using the product and quotient rules. Ex. 3–30.
  - Understanding the product and quotient rules. Ex. 43–50.
- **Section 3.3**
  - * Find derivatives of trigonometric functions. Ex. 1–25.
  - Finding limits involving trigonometric functions. Ex. 45–55.
- **Section 3.4**
  - * Use the chain rule to find derivatives of composite functions. **This is important!** Ex. 7–51, 50, 52.
  - Understand the chain rule. Ex. 76–78.
- **Section 3.5**
  - * Find the derivative of an implicitly defined function. Ex. 5–22.
  - * Find the equation of the tangent to the curve given by an equation. Ex. 27–33. Ex. 53, 55, 62.
- **Section 3.6**
  - * Find the derivative of functions involving logarithms. Ex. 2–16, 27, 28.
  - Use logarithmic differentiation when needed to find the derivative of some functions. Ex. 49–56.
  - Find derivatives of inverse trigonometric functions. Ex, 63–72.
- **Section 3.8**
  - Understand exponential growth and decay, and how it is expressed using derivatives (the instantaneous rate of change of some quantity is proportional to the existing quantity). Ex. 3, 8, 9, 11.
- **Section 3.9**
  - Understand that when there is a relation between two or more quantities that are changing, by taking derivatives one obtains a relation between the instantaneous rate of change of the quantities.
  - Apply this to some concrete examples. Ex. 3, 5, 7, 14, 17, 53.
- **Section 3.10**
  - * Understand that the tangent line to a graph at a point gives the best linear approximation to the function near that point.
  - * Find the linear approximation (or linearization) of a function f at a point a (given by $L(x) = f(a) + f'(a)(x - a)$). Ex. 1–4.
  - Use the linearization to find the approximate value of a function. Ex. 31–36.
  - Understand the meaning of differential and compute the differential of a function. Ex. 11–16.

- **Section 4.1**
  - * Understand the concept of absolute and local maxima and minima. Ex. 3–8.
  - The critical numbers (or points) of a function $f(x)$ are the values of x where the derivative is 0 or where the derivative does not exist. Find critical numbers: Ex. 29, 30, 35, 37, 41, 43.
  - * Find the absolute maximum and minimum values of a function in an interval. Ex. 51–66.
  - Understand that the local maxima and minima of a function only occur at the critical numbers.
- **Section 4.2**
  - * Understand Rolle's Theorem and the Mean Value Theorem. It is important that you know the picture that explains the theorem. Ex. 1, 3, 4.
  - Apply the Mean Value Theorem to conclude properties of functions. Ex. 29, 30, 31.
- **Section 4.3**
  - * Understand what it means for a function to be increasing or decreasing, and concave up or concave down, in an interval. Ex. 1, 2, 3, 34, 35.
  - * Remember that $f'(x) > 0$ implies that f is increasing at the point x, and $f'(x) < 0$ implies that f is decreasing at the point x. Use this fact to find the intervals of increase and decrease of f. Use this fact to identify whether a critical point is a local maximum or a local minimum. Ex. 8-16.
  - * Remember that $f''(x) > 0$ implies that f is concave up at the point x, and $f''(x) < 0$ implies that f is concave down at the point x. Use this fact to find the intervals of concavity and convexity of f. Ex. 17–21, 23, 25.
- **Section 4.4**
  - * Understand L'Hopital's rule and apply it to find limits. Ex. 8–20.
  - Identify the different "indeterminate forms" of limits, and convert them to limits that can be calculated using L'Hopital's rule. Ex. 43, 44, 47, 51, 53, 55, 57, 59, 61.
- **Section 4.5**
  - * Given a function, know how to find its domain, intercepts, asymptotes, intervals of increase and decrease, local max and min, concavity, inflection points. Ex. 1–4, 9–14, 23, 45, 46.
  - * Given the information from the previous item, know how to interpret it an sketch the graph of the function. (Same exercises as last item.)
- **Section 4.7**
  - * Given an optimization problem, know how to set up the problem to write the quantity to be optimized as a function of a single variable. Ex. 3–7, 25–27, 60, 61
- **Section 4.8**
  - * Understand the picture for Newton's Method to approximate zeros of functions. Ex. 1–5.
  - Use Newton's Method. Ex. 17, 18.

- **Section 4.9**
  - * Understand the concept of antiderivative (or indefinite integral) and know how to find the antiderivative of basic functions (table of antidifferentiation formulas in this section). Ex. 1–20, 27–32.
  - Understand the relationship between position, velocity, and acceleration of an object moving in a straight line. Ex. 65–68.
- **Section 5.1**
  - * Understand the definition of area under the graph of a positive continuous function using approximating rectangles (Riemann sum). Ex. 3, 4, 17, 19.
- **Section 5.2**
  - * Understand the definition of the definite integral (which is the same as the area from section 5.1 for positive functions). Ex. 1, 27, 28.
  - Understand and be able to use the properties of the indefinite integral. Ex. 51–57.
- **Section 5.3**
  - * Understand and be able to use the Fundamental Theorem of Calculus, part 1. Ex. 9–14, 67–70.
  - * Understand and be able to use the Fundamental Theorem of Calculus, part 2. Ex. 25–34
- **Section 5.4**
  - * Definition of indefinite integral (same thing as antiderivative). Know the table of indefinite integrals (just remember that you just have to find a function whose derivative is the integrand). Ex. 1–3, 5–14.
  - * Use indefinite integrals to calculate definite integrals. Ex. 27–34.
  - Understand the statement of the net change theorem (which is just a restatement of the Fundamental Theorem of Calculus, part 2): the integral of the rate of change is the net change. Ex. 59–61, 73, 74.
- **Section 5.5**
  - * Know how to make a substitution to compute an indefinite integral when the change of variable is given. Ex. 1–8.
  - Know how to find the appropriate substitution to compute an indefinite integral. Ex. 9–20, 59–66.