6.4. General strategy for factoring polynomials. Professor Luis Fernandez

To factor a polynomial, here is a summary of the steps and methods:

1. **Factor out the Greatest Common Factor (GCF)**
   - First find the GCF of the terms of the polynomial:
     - Find the GCF of the coefficients.
     - Find the GCF for each variable. If a given variable appears in all the terms, the GCF for that variable is the variable raised to the lowest exponent that it has in the terms. If not, the GCF for that variable is 1.
     - The GCF is the product of the GCF of the coefficients and the GCF for each variable.
   - Write the GCF out of the parenthesis.
   - Inside the parenthesis, divide the polynomial by the GCF.

   Once the GCF has been factored out you will have a product of polynomials. For each one of these polynomials, count how many terms the polynomial has. If it is a

2. **Monomial (1 term).**
   - Monomials are already factored.

3. **Binomial (2 terms).**

   QUESTION: **Is it is a difference of squares?**
   - YES $\to$ Factor using $a^2 - b^2 = (a+b)(a-b)$.
   - NO $\to$ Cannot be factored further with the methods you know.

4. **Trinomial (3 terms).**

   QUESTION: **Is it a perfect square?** (that is, does it have the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$)?
   - YES $\to$ Factor using $a^2 + 2ab + b^2 = (a+b)^2$ or $a^2 - 2ab + b^2 = (a-b)^2$.
   - NO $\to$ Try the next method:

   QUESTION: **Is it monic?** (that is, is the leading coefficient equal to 1?)
   - YES $\to$ Then the polynomial will have the form $x^2 + bx + c$.
     - Identify the numbers b and c.
     - Find two numbers m and n such that
       $m \cdot n = c$
       $m + n = b$
     - If you find the two numbers m and n, then $x^2 + bx + c = (x+m)(x+n)$.
     - If after checking all the possibilities no such numbers exist, the polynomial cannot be factored with the methods you know.
   - NO $\to$ Then the polynomial will have the form $ax^2 + bx + c$. Use the ac-method:
     - Identify the numbers a, b and c.
     - Find two numbers m and n such that
       $m \cdot n = a \cdot c$
       $m + n = b$
     - If you find the two numbers m and n, then
       * Since $m+n = b$, break the middle term bx of the polynomial as $mx+nx$ to get $ax^2+mx+nx+c$, which has 4 terms.
       * Factor the polynomial by grouping (see below).
     - If after checking all the possibilities no such numbers exist, the polynomial cannot be factored with the methods you know.

5. **Polynomial with 4 terms.**

   Try factorization by grouping:
   - Factor out the GCF of the first two terms and the GCF of the last two terms separately. You will have a polynomial with only two terms now.
   - QUESTION: **Do the two terms have a common factor?**
     - YES $\to$ Factor out the common factor of the two terms.
     - NO $\to$ The polynomial cannot be factored with the methods you know.

Once you have finished, always check your work by multiplying out the factorization you got.

Practice exercises: Factor the following polynomials completely.

1. $2n^2 + 13n - 7 =$

2. $49b^2 - 112b + 64 =$

3. $75x^3 + 12x =$

4. $4x^5y - 32x^2y =$

5. $x^4 - 81 =$

6. $12ab - 6a + 10b - 5 =$

7. $16x^2 - 24xy + 9y^2 - 64 =$

8. $60x^2y - 75xy + 30y =$

9. $3x^4 - 48 =$

10. $9x^2 - 12xy + 4y^2 =$

11. $9 + 30x + 25x^2 =$

12. $64x^2 + 112xy + 49y^2 =$

13. $x^4 - 13x^3 + 42x^2 =$

14. $20x^3 + 40x^2 + 15x =$

15. $48x^3 - 12x =$

16. $x^4 - y^4 =$

17. $7x^2 - 35x + 42 =$

18. $x^2 + 1 =$