MTH 42 LECTURE NOTES (Ojakian)

Topic 16: Markov Chains and Equilibrium

OUTLINE
(References: 3.5)

1. Markov Chains

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1. First Example

PROBLEM 1. *Consider NYC and imagine we put everyone in a cateogory of Democrat, Republican, or Independant.*

   (a) Pick a percentage of the population for each category.
   (b) Make up a graphical represenation of how the voters change by year.
   (c) Represent this graphical representation as a matrix.
   (d) Figure out how the percentages change in one year.
   (e) Notice how figuring out the percentage changes is the same as matrix multiplication
   (f) In Anaconda, using matrix multiplication, observe what happens as the years pass (and note that this process can be simplifid with matrix algebra).
   (g) In Anaconda, find the eigen values and eigen vectors. Anything surprising?

2. Markov Chain Terminology

   (a) Initial Vector
   (b) Probability Vector
   (c) Stochastic Matrix
   (d) Steady State Vector (i.e. vector with eigen value = 1!)
   (e) Markov Process: Start with initial probability vector and repeatedly apply the transition matrix.
   (f) Regular Matrix

3. Markov Chain Theorems

   (a) See Theorem 3.27 (page 144)
   (b) See Theorem 3.29a (page 146)

   PROBLEM 2. *Experiment in Anaconda with different initial vectors and different matrices to illustrate the last theorem.*

   PROBLEM 3. *Show the work to actually find the steady state vector by hand (though we might let Anaconda help for an involved calculation).*

4. Group Work: Make your own Markov Chain!