MTH 42 LECTURE NOTES (Ojakian)

Topic 15: Eigen Vectors and Eigen Values

OUTLINE
(References: 6.1)

1. Eigen Vectors
2. Eigen Values

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1. Definitions of Eigen Vector and Eigen Value

   (a) See Definition 6.1

   (b) Recall Leontieff's Closed Economy - production vector was an eigen vector.

   (c)

   PROBLEM 1. *Do exercise 1 from Section 6.1 (page 227).*

   PROBLEM 2. *Find a 3 by 3 matrix such that every non-zero vector is an Eigen Vector.*

   (d) See Theorem 6.2: Once you have one Eigen Vector, you have more Eigen Vectors.

2. Finding Eigen Vectors and Eigen Spaces

   (a) Finding Eigen Vectors when given an Eigen Value

   PROBLEM 3. *Consider exercise 11 in Section 6.1 (page 228). Assume you have magically been given an eigen value.*

   i. Find the associated eigen vectors.

   ii. Describe these eigen vectors as the span of some vectors (as long as you throw in the zero vector).

   iii. Conclude that this set of eigen vectors is a subspace and find its dimension.

   (b) Eigen Space

   See Theorem 6.3 and Definition 6.4

   PROBLEM 4. *Note that in the last problem we found an eigen space.*

3. Finding Eigen Values

   (a) See Theorem 6.5.

   Note: Characteristic Polynomial and Characteristic Equation.

   PROBLEM 5. *Do exercise 21 from Section 6.1 (page 228) - just find the eigen-values*

4. <u>Putting it altogether</u>

PROBLEM 6. *Continue the last problem - find the eigen spaces associated to each eigen value.*

(a) Multiplicity of Eigen Value

   i. Definition of multiplicity for a root of a polynomial.

   PROBLEM 7. *Find all the roots and multiplicities of the following polynomial:* $y^3 - 6y^2 + 9y$

   ii. See Theorem 6.6

   PROBLEM 8. *Consider the last problem. Suppose that polynomial is the characteristic polynomial for some matrix. Determine the maximum possible dimensions of each eigen space.*

   iii.

   PROBLEM 9. *Do exercise 29 from Section 6.1 (page 228) - note the validity of Theorem 6.6*

   iv.

   PROBLEM 10. *Do exercises from Section 6.1 (pages 228-229): 37, 47.*

(b) Why using characteristic polynomial works?

   i. Recall key fact about determinant related to invertibility

   ii. Note the fact: $M\bar{x} = \bar{b}$ has a unique solution **if and only if** M is invertible.