MTH 42 LECTURE NOTES (Ojakian)

Topic 13: Row and Column Spaces

OUTLINE (References: 4.3)

1. Row and Column Spaces

2. Rank of a matrix

3. Rank-Nullity Theorem

1. Row space and Column space and Rank

(a) See Definition 4.19.

PROBLEM 1. Do exercise 1 on page 178 (wait on Rank-Nullity).

PROBLEM 2. How does the dimension of the row space relate to the dimension of the column space in the last problem? In general? Justify your answer and note Theorem 4.21

(b) Definition 4.22

PROBLEM 3. For exercise 1 above, what is the rank of the matrix?

- (c) Note: Column space = Range of corresponding linear transformation.
- (d) Note: Row space does not correspond to something we have talked about (I think!)

2. Rank-Nullity Theorem

(a) Def: "Nullity" is the dimension of the Null Space

PROBLEM 4. For exercise 1 above, what is the nullity of the matrix? **PROBLEM 5.** For exercise 1, based on the rank and nullity, can you guess a theorem?

- (b) Theorem 4.23
- (c) Restated:

Dimension of Null Space + Dimension of Range Space = Number of Columns. **PROBLEM 6.** Do exercises 13, 15, 17 on pages 178-179.

PROBLEM 7. Do exercise 7 on page 178.