MTH 42 LECTURE NOTES (Ojakian)

Topic 12: Basis and Dimension

OUTLINE

(References: 4.2)

1. What a basis is.
2. Key properties.
3. Two ways to find a basis.
4. Dimension

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1. Definition of Basis
   (a) See Definition 4.8
   (b) Key Fact: Theorem 4.9.
   (c)
   PROBLEM 1. *Do section 4.2: 1, 2, 3.*
   If it is a basis, verify Theorem 4.9.
   PROBLEM 2. *Find two different examples of a basis for R^3. Verify Theorem 4.9 for at least one of these.*
   Also find examples of sets of vectors in R^3 which are **not** *a basis.*
   PROBLEM 3. *Consider the subspace S of R^3 which consists of the x-y plane. Find two different examples of a basis for S.*

2. Dimension
   (a) Theorem 4.12 and Definition 4.13 and not dimension of $\{\vec{0}\}$
   PROBLEM 4. *Find the dimension of R^3. Find the dimension of R^n for any n.*
   (b) Note the significance!
   Theorem 4.9 together with Theorem 4.12

3. Finding a Basis: The row approach

   Given a set of vectors, find a basis of their span, *not necessarily using any of those vectors.*

   (a) Make the vectors into the rows of a matrix.
   (b) Put the matrix into echelon form.
   (c) The non-zero rows are a basis.
   (d)

   PROBLEM 5. *Let S be the span of* $(1,2,0,1),(3,7,1,0)$*, and* $(7,11,1,2)$*. Find a basis of S.*

   (e) Why does this work?
      i. Row operations do not change the span of the rows.
      ii. Rows of Echelon Form matrix are linearly independent.

4. Finding a Basis: The column approach

   Given a set of vectors, find a basis of their span, *among those vectors.*

   (a) Make the vectors the columns of a matrix.
   (b) Put the matrix into echelon form.
   (c) The columns of the **original matrix** corresponding to the pivot columns are a basis.
   (d)

   PROBLEM 6. *Do problem 5 again, but this time find a basis contained in the original vectors.*

   (e) Why does this work?
      i. Row operations do not change the dependencies among the columns.
      ii. For a matrix in Echelon form, the set of pivot columns are linearly independent.
      iii. For a matrix in Echelon form, any non pivot column is a linear combination of the pivot columns.