MTH 42 LECTURE NOTES (Ojakian)

Topic 11: Subspaces

OUTLINE
(References: 4.1)

1. Subspaces
2. Null Space = Kernal
3. Range Space

---

1. Subspace Definition

(a) Closed under vector addition.

(b) Closed under scalar multiplication

(c) Definition of subspace: Non-empty, closed under vector addition and scalar multiplication (slight modification of Definition 4.1 which is equivalent).

(d) Subspace or not subspace? (First, just in R^2!)

i. Directly describing set of points

PROBLEM 1. *Consider the line segment in R^2 connecting the two points $(-1,-2)$ and $(1,2)$. Is this set of points a subspace or not?*

ii. Describing points as the set of solutions to an equation

PROBLEM 2. *Consider the line in R^2 given by the equation $y = 2x + 1$. Is this set of points a subspace or not?*

PROBLEM 3. *Consider the line in R^2 given by the equation $y = 2x$. Is this set of points a subspace or not?*

iii. A generic description of a set of points

PROBLEM 4. *Consider any 2 dimensional ball of finite radious in R^2. Is this set of points a subspace or not?*

iv. Describe them all?

PROBLEM 5. *Describe all the subspaces of R^2.*

2. Subspace Facts

(a) Points to check

i. Useful for checking if a set is NOT a subspace.

Theorem 1. *A subspace always contains the zero vector.*

ii. Useful for checking if a set is IS a subspace: Theorem 4.2

iii. Consider checklist on page 153.

(b) More Examples

i. Describing a set of vectors with mathematical notation

PROBLEM 6. *Do exercise 1 on page 158.*

PROBLEM 7. *Let* $X = \{(2s, 5s, -s) \mid \text{ for all } s \text{ in } R\}$. *Is* X *a subspace of* R^3.

PROBLEM 8. *Describe geometrically the subspaces in the last two problems. Describe all the subspaces of* R^3.

ii. Describing a set of vectors as the solution to a system of equations

PROBLEM 9. *Pick some non-homogeneous system of equations. Is its set of solutions a subspace?*

PROBLEM 10. *Pick some homogeneous system of equations. Is its set of solutions a subspace?*

iii. Define Null Space: See definition 4.4 on page 155.

3. Important subspaces: Null Space, Kernal Space, and Range Space

(a)

PROBLEM 11. *Consider exercise 27 on page 159. Do the following:*

i. Write down the linear transformation T *that is defined from the matrix* A.

ii. Find the Null Space for A.

iii. Describe the set of vectors $\bar{x}$ *such that* $T(\bar{x}) = \bar{0}$.

iv. Describe the Range of T. *Is it a subspace?*

(b) Define: Kernal Space and Range Space

PROBLEM 12. *Do exercise 34 on page 159 (use Anaconda).*

PROBLEM 13. *Explain/prove that the Range Space is a subspace.*

Note: Theorem 4.5 on page 156. Notice which R^x each is a subspace of!

PROBLEM 14. *Make up a linear transformation. Determine which* R^x *each is a subspace of: Kernal Space, Null Space, Range Space*

(c) One-to-One Theorem

Note: Theorem 4.6 on page 157

PROBLEM 15. *Consider exercise 27 on page 159 again. Is the linear transformation defined from this matrix injective? Do this in two ways: using linear independence and Theorem 4.6*