MTH 42 LECTURE NOTES (Ojakian)

Topic 10: Leontief Economic Model

OUTLINE

(References: Supplemental Lay 2.6)

1. Closed Model
2. Open Model

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1. Leontieff

   Background on person and discovery ...

2. Basics

   (a) Consumption vector, inputs/outputs, and net output.

      i. Consumption vector: Required inputs needed to produce \$1 of output.

      PROBLEM 1. *Do examples of some consumption vectors (some reasonable and some pathological). Determine the Inputs needed for various Outputs. Find the Net Output in dollar value.*

   (b) Consumption Matrix

   Example:

$$\begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.6 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.2 \end{bmatrix}$$

   PROBLEM 2. *Do examples of some consumption matrices (some reasonable and some pathological), starting with the above one, just the meaning as you read column by column.*

   (c) What is the meaning of: A consumption matrix times a vector?

   The vector represents: Output Production Goal

   The result of the multiplication, represents: Input Production Demand.

   PROBLEM 3. *For the above consumption matrix, consider the meaning of various multiplications. Consider the meaning if the matrix is changed.*

3. Closed Economy

   All demand from from the producing sectors (more on this later ...)

   PROBLEM 4. *Consider the example of an economy with 3 sectors: Manufacturing, Agriculture, and Services, with the consumption matrix given as follows (sectors in order).*

$$\begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.6 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.2 \end{bmatrix}$$

   Do the following:

(a) *Discuss the meaning of the matrix (in dollar values), and the prime goal: Production exactly equals demand.*

(b) *Show why the production level (10, 2, 2) does NOT work.*

(c) *Set up and understand the Matrix Equation that completely answers the question.*

(d) *Derive and use the following matrix equation:* $(I - C)\bar{p} = \bar{0}$*; solve using Anaconda. item Find Net Production = Output Production - Input Production =* $\bar{p} - C\bar{p}$.

4. Open Economy

(a) Oddity about above example (called Closed Economy): There are no consumers! Just a bunch of producers, producing for oneanother.

PROBLEM 5. *To be concrete, in the above example recall the net production of each sector. Also, just by looking at the consumption matrix, how much money do you need to produce one $1 of anything? What is odd about that?*

PROBLEM 6. *Consider the same 3 sectors as above, but now use the following consumption matrix:*

$$\begin{bmatrix} 0.5 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix}$$

i. *How much money is needed to produce a $1 of any commodity?*

ii. *Suppose you want output production of (10, 5, 5). What input production do you need? What is your net production?*

(b) Open Economy: Have excess which can go to consumers, as in the last example!

(c) Define: External Demand (or Final Demand), $\bar{d}$ - i.e. from non-producing consumers (when $\bar{d}$ is not the zero vector, we have an **Open Economy**).

(d) Define: Internal demand (or: Intermediate Demand, or: Input Production Demand)

(e) Derive equation: $\bar{p} = C\bar{p} + \bar{d}$ as follows:

i. From above: $\bar{p}$ is Output Production Goal

ii. So $C\bar{p}$ is Input Production Demand

iii. So $\bar{p} - C\bar{p}$ is leftover to give to consumers.

(f) Solve the above for $\bar{p}$ to get: $(I - C)\bar{p} = \bar{d}$

(g) Theorem (Theorem 11 in Lay): If a consumption matrix has all non-negative entries, where the column sums are all less than 1, and we are in an open economy, then there is a unique non-zero solution for the production level. Furthermore, the matrix $(I - C)$ is inveritble.

PROBLEM 7. *Consider the same 3 sectors as above, but now use the following consumption matrix:*

$$\begin{bmatrix} 0.5 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix}$$

i. *Pick an external demand vector which makes this an open economy.*

ii. *Verify that the conditions in the above theorem are true. What does the theorem guarantee?*

iii. Solve the system in TWO ways using Anaconda:

A. Solve using Gauss-Jordan Elimination.

B. Solve by finding inverse of $(I - C)$.

5. <u>Pathological Examples</u>

(a) Consider how much input sector 3 needs to produce a given amount of output.

$$\begin{bmatrix} 0.5 & 0.4 & 0 \\ 0.2 & 0.3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) In the following example, consider an external demand of $(100, 200)$. Intuitively, what about this economy leads to no reasonable solution for production? (Try it in Anaconda)

$$\begin{bmatrix} 0.06 & 0.10 \\ 0.05 & 1.02 \end{bmatrix}$$

(c) By reversing, where the large number is, consider how the following economy makes sense. (Try it in Anaconda).

$$\begin{bmatrix} 0.06 & 1.02 \\ 0.05 & 0.10 \end{bmatrix}$$