MTH 42 LECTURE NOTES (Ojakian)

Topic 8: Matrix Algebra

OUTLINE

(References: 3.2)

1. Matrix operations
2. Properties of Matrix Multiplication

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1. <u>Matrix Addition and Scalar Multiplication</u>

   Do example and see calculations in Anaconda.

   Note Theorem 3.11.

2. <u>Matrix Multiplication</u>

   (a) It is **not** what you might think!

   PROBLEM 1. *Evaluate $A \cdot B$, where A is a 2 by 2 matrix and B is a 2 by 3 matrix. Use both definitions below.*

   (b) Definition 1 (Definition 3.12) - the "vector arithmetic approach".

   In $A \cdot B$, each column of B gives a linear combination of the A columns (generalizes: Matrix times Vector)

   (c) Definition 2 (Figure 2 on page 99) - the "dot product approach"

   i. Define the dot product of two vectors.

   ii. Do Row i of A DOT Column j of B to get entry (i, j) in the product.

3. <u>Matrix Properties</u>

   (a) What **IS** true: Theorem 3.13.

   PROBLEM 2. *Pick a property and verify (not proof!) it for an example.*

   (b) What is **NOT** true! Do and discuss Theorem 3.14

   PROBLEM 3. *Find examples to make each true - first because of matrix dimension problems, and then because of the matrix content.*

   (c) Transpose of a Matrix.

   PROBLEM 4. *Consider any matrix and find its transpose.*

4. Why Matrix Multiplication Defined As It Is?

   One good reason: Function composition!

   PROBLEM 5. *Make up a linear transformation* $T_1 : R^3 \to R^2$ *and a linear transformation* $T_2 : R^2 \to R^4$. *Then do the following:*

   (a) Calculate $T_2 \circ T_1$ *on some inputs.*

   (b) Find the matrix representing T_1 *and* T_2.

   (c) Find the matrix representing $T_2 \circ T_1$. *And check your inputs from the first part with this matrix.*

5. Matrix Powers and Networks

   Theorem 1. *Let* A *be the adjacency matrix of a graph. Then for integers* $k \geq 1$, *entry (i,j) of* A^k *is the number of walks between vertex* i *and vertex* j.

   PROBLEM 6. *Consider a graph which is a path of length 3. Find its adjacency matrix and see how its matrix powers fit with the above theorem. Experiment in Anaconda to see higher matrix powers.*