MTH 42 LECTURE NOTES (Ojakian)

Topic 6: Span and Linear Independence

OUTLINE
(References: 2.2, 2.3)

1. Span
2. Linear Independence

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1. Definition of Span
   (a) Examples in R^2 and R^3.

   PROBLEM 1. *Find some vectors in the span of some other vectors, by hand and using Anaconda.*

   (b) Definition 2.5

   (c)

   PROBLEM 2. *In R^3 is $(1,0,0)$ in the span of $\{(2,1,1),(1,2,3)\}$?*
   Note: Convert **Vector Equation** *to* **System of Equations**.

   PROBLEM 3. *Is the vector $(6,4)$ in span$\{(0,1),(2,1)\}$?*

   PROBLEM 4. *Show that all the vectors of R^2 are in span$\{(0,1),(2,1)\}$.*

2. How many vectors needed to span R^n?
   (a) We have seen that n vectors can span R^n. Do n vectors always span R^n?
   (b) For every n, are there some n vectors that span R^n?
   (c) Can less than n vectors span R^n?

   PROBLEM 5. *From Section 2.2, page 66 - Do exercise 37.*
   *Then see how this is a "***proof by example!?***" of Theorem 2.8 on p. 63*

   PROBLEM 6. *From section 2.2 (page 67): Exercise 65.*

3. Matrix Equation and Converting between forms
   (a) Matrix Form: See Definition 2.9

   PROBLEM 7. *From Section 2.2 (page 64): Do Example 7.*

   (b) System of Equations TO Matrix Form

   PROBLEM 8. *From Section 2.2 (page 65) do: 13*

4. Redundancy and Linear Independence

(a) Do Theorem 2.7 by example in R^2.

(b) Two equivalent ways to talk about one idea:

i. "Linear Independence" or

ii. "Linear Dependence"

Definition 1. *(Defined in termed of linear dependence - Theorem 2.14)*
*A set of vectors in R^n is **linearly dependent** if any one vector is in the span of the others.*
*A set of vectors is **linearly independent** if they are not linearly dependent.*

Definition 2. *(Defined in terms of linear independence - Definition 2.11)*
*A set of vectors $\{u_1, \ldots, u_m\}$ in R^n is **linearly independent** if the only solution to the equation*

$$x_1u_1 + \cdots + x_mu_m = 0$$

is $x_1 = \cdots = x_m = 0$.
*A set of vectors is **linearly dependent** if they are not linearly independent.*

(c) Checking if a set of vectors is linearly independent
Note: You do NOT need to solve the system. Just determine if it has a non-trivial solution or not!

PROBLEM 9. *Do section 2.3 (page 77): Exercises 3 and 10.*
(By hand and with Anaconda)

(d) Find 3 vectors in R^2 that are linearly independent?

(e) **Discuss Theorem 2.13.**

PROBLEM 10. *From Section 2.3 (page 79): Do exercise 53.*

5. Homogenous system and non-Homogenous system

Example 5.

Theorem 2.17

PROBLEM 11. *Suppose the system $Ax = 0$ has the general solution:*

$$(3s - 2t, s, 4t, t).$$

And suppose that $Ax = b$ has $(1, 0, -2, 7)$ as one of its solutions. Express the general solution to the system $Ax = b$.

6. The Big Theorem

Discuss Theorem 2.19

7. Some Proofs

PROBLEM 12. *Section 2.2 (page 67): Exercises 67, 71*

PROBLEM 13. *Section 2.3 (page 79): Exercises 61*

PROBLEM 14. *Section 2.3 (page 79): Exercises 58 and 59*