MTH 42 LECTURE NOTES (Ojakian)

Topic 5: Vectors

OUTLINE
(References: 2.1)

1. Vectors in Euclidean Space
2. Operations on vectors and proving properties
3. Expressing things with vectors
4. Geometry of vectors

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1. Euclidean Space: Vectors in R^n
   (a) Column vector versus row vector
   (b) Why? - One reason: a vector is a solution to a system
2. Operations on Vectors
   (a) Addition
   (b) Scalar multiplication (note: we do **not** define multiplication of vectors here).

   PROBLEM 1. *If* $v = (4, 0, -2, 1)$ *and* $w = (1, -1, 3, 0)$ *evaluate the following:*

   i. $3 \cdot v$
   ii. $w + v$
   iii. $2v - w$

   (c) Properties of these operations: See Theorem 2.3 and note the following terminology:
      i. Zero Vector
      ii. (Additive) Inverse of a vector
      iii. Properties: Commutativity, Associativity, Distributivity
   (d)

   PROBLEM 2. *Prove property (b) of Theorem 2.3*

3. Expressing things with vectors

(a) Define: Linear combination of vectors.

(b) Different ways to express the *problem.*
Example: Pick one and translate to other

i. System of Equations

ii. Augmented Matrix

iii. Vector Equation

PROBLEM 3. *From Section 2.1 (page 54): Do exercise 9. Also write it as an augmented matrix.*

PROBLEM 4. *Express the following system of equations as a vector equation:*

$$\begin{cases} 2x_1 + x_2 - x_3 = 2 \\ 6x_1 + 3x_3 = 6 \end{cases}$$

(c) Different ways to express the *solution.*

i. General solution (with variable names)

ii. Vector solution

PROBLEM 5. *From Section 2.1 (page 54) do exercise 17.*

(d) Combining compound objects.

PROBLEM 6. *From Section 2.1 (page 55) do exercise 45 (at least the set up).*

4. Geometry of vectors in R^2

(a) Drawing from tail to tip.

(b) Geometry of scalar multiplication:
Consider multiplication by positive (more and less than 1), by negative, by zero.

(c) Geometry of adding: Tip-to-Tail rule or Parallelogram Rule

PROBLEM 7. *From Section 2.1 (page 57) do exercise 77.*

(d) Geometry of subtraction: Draw vector towards the first vector

PROBLEM 8. *From Section 2.1 (page 57) do exercise 79.*
Also, do the subtraction algebraically then draw the resulting vector.

Note: To justify this, for $v - w$, apply the parallelogram rule to $v + (-w)$.