MTH 42 LECTURE NOTES (Ojakian)
Topic 3: Gaussian Elimination

OUTLINE
(References: 1.2)

1. Matrix forms: Echelon and Reduced Echelon

2. Gaussian Elimination and Gauss-Jordan Elimination

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1. Gaussian Elimination on Systems

   Putting system into Echelon Form.

   (a) Note Elementary Operations on a system:

   PROBLEM 1. *Do Section 1.2 - Exercise 21 (page 27) - Just as a system of equations.*

2. Converting System to Matrix

   PROBLEM 2. *Convert the system from Problem 1 into a matrix.*

   Note **Augmented Matrix** versus **Coefficient Matrix**.

3. Gaussian Elimination (on matrices)

   (a) Note Elementary Row Operations

   PROBLEM 3. *Redo Problem 1, but now by converting it to a matrix first.*

   (b) Echelon Form - see definition 1.4.

   (c) Terminology

      i. Leading entry (or leading term) (of a row): its leftmost **value**
      ii. Pivot position: A **position (or location)** that contains a leading term.
      iii. Pivot: A **number value** in a pivot position.
      iv. Pivot column: A **column** that contains a pivot position.

4. Gauss-Jordan Elimination

   PROBLEM 4. *Do Section 1.2 Exercise 1.2 - 29 (page 27). Note: Forward phase and backwards phase.*

   (a) Note Definition 1.5 (**Reduced** Echelon Form)

   (b) Interesting point (page 23): Theorem 1.6

   Note true or Echelon form (only **reduced** Echelon form)!

5. <u>Theoretical Observations</u>

    (a) Information from the shape:

        PROBLEM 5. *Consider an augmented matrix representation of a system of equations. Suppose it has been put into Echelon Form. What do the following conditions imply, if anything? (consider each one independent of the others).*

          i. *A row with all zeros, except for the rightmost entry, which is not a zero.*
          ii. *A row with all zeros.*
          iii. *The rightmost column is a pivot column.*
          iv. *The rightmost column of the coefficent matrix is a pivot column.*
          v. *The system is triangular.*
          vi. *The system is not triangular.*
          vii. *The system is not triangular AND has no pivot in the rightmost column.*

    (b) Do the proof of Theorem 1.2 (page 25) : It incorpoates the ideas of the above problem.

    (c)

        PROBLEM 6. *Do Section 1.2 Exercises - 37, 51 (page 27).*