# MTH 42 LECTURE NOTES (Ojakian)

## Topic 1: Systems of Linear Equations

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**OUTLINE**
(References: 1.1)

1. Linear Equations
2. Systems of Equations
3. Lines and Planes

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1. Linear Expression

   Examples, using variables $x_1, x_2, \ldots$

2. Linear Equation

   (a) What it is.

   i. Examples with 2 variables: Lines on the plane.
   ii. Examples with 3 variables: Planes in $R^3$.
   iii. Examples with more variables.
   Definition: In general, a linear equation in $n$ variables gives a Hyperplane in $n$-dimensional space.

   (b) Aside: What is $R^n$?

   (c) Solution Set

   **PROBLEM 1.**

   - *Find 2 different solutions to the equation and graph it:* $3x + 4y = 1$.
   - *Find the* **General Solution**, *using a* **Free Parameter**.

   **PROBLEM 2.**

   - *Find 3 different solutions to the equation and graph it:* $x + y + z = 1$.
   - *Find the General Solution.*

3. Systems of Linear Equations

   (a) What it is.

   i. Examples with 2 variables (see below).
   ii. Examples with 3 variables (see below).
   iii. Examples with more variables.

   (b) Solution Set

   i. Terminology: **Consistent**, **Inconsistent**
   ii. Examples with 2 variables: Intersection of lines.

   **PROBLEM 3.** *Solve the system by graphing:* $\begin{cases} x_1 + x_2 = 0 \\ 3x_1 + x_2 = 0 \end{cases}$

**PROBLEM 4.** *Solve the system by graphing (***use computer***):* $\begin{cases} 2x_1 + x_2 = 2 \\ 6x_1 + 3x_2 = 2 \end{cases}$

iii. Examples with 3 variables: Intersection of planes.

**PROBLEM 5.** *Solve the system by .. thinking:* $\begin{cases} x_1 + x_2 + 2x_3 = 0 \\ 2x_1 + 2x_2 + 4x_3 = 0 \end{cases}$

**PROBLEM 6.** *Solve the system by graphing using computer (and thinking):*

$$\begin{cases} x_1 + x_2 + 2x_3 = 0 \\ 2x_1 + 2x_2 + 4x_3 = 9 \end{cases}$$

iv.

**PROBLEM 7.** *Conceptually, what can a solution set look like for the following scenarios.*

*A. A system of equations with 3 variables and 2 equations.*

*B. A system of equations with 3 variables and 3 equations.*

*C. A system of equations with 3 variables and 100 equations.*

*D. See the textbook picture on pages 6 and 7.*

v. Examples with more variables. Not to solve yet!

4. A First Theorem

**PROBLEM 8.** *Consider some system of linear equations. Based on the examples, what sizes are possible for the solution sets?*

*(Answer in Theorem 1.2; don't peak!)*