

MTH 32 LECTURE NOTES (Ojakian)

Topic 15: Conic Sections

OUTLINE

(References: 7.5)

1. Fundamental conic section definitions
 2. Graphing conic sections
 3. General formulas, properties of conic sections
 4. Applications of conic sections
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1. Conic Sections

- (a) Collections of points (Mostly Funny Ones ...)

PROBLEM 1. *Given a point and a number K , describe the set of points distance K from that point.*

PROBLEM 2. *Given two distinct parallel lines, describe the points that are equidistant from the two lines.*

***PROBLEM* 3.** *Given a line and a number K , describe the set of points distance K from that line.*

***PROBLEM* 4.** *Given two distinct points, describe the set of points equidistant from the two points.*

***PROBLEM* 5.** *(Extra For Fun!) Suppose you have a line and a point a distance K from the line. Describe the points with the following property: the sum of the distances from the point and the line is K .*

- (b) Important collections of points!

***PROBLEM* 6.** *Given a line and a point not on the line, describe the set of points that are **equidistant** from the line and the point.*

***PROBLEM* 7.** *Given two distinct fixed points and a number K , describe the set of points with the following property: the **sum** of the distances to these two fixed points is K .*

***PROBLEM* 8.** *Given two distinct fixed points and a number K , describe the set of points with the following property: the **difference** of the distances to these two fixed points is K .*

- (c) Name the three above curves

- (d) Show how to get the 3 curves as “Conic Sections”

2. Parabolas

- (a) Theory

Typical form of parabola: $y = ax^2$ or $x = ay^2$

Formula (for focus): $p = \frac{1}{4a}$ 1

PROBLEM 9. Find the vertex, focus and directrix of the parabola $4x^2 + 8y = 0$.

***PROBLEM* 10.** From WORK BOOK, section 27, do problem 4.

(b) Applications

A parallel ray reflects to the focus (and reverse from the focus).

PROBLEM 11. From Textbook, section 7.5: Problems 312, 313.

***PROBLEM* 12.** From Textbook, section 7.5: Problems 314.

3. Ellipses

(a) Typical form of ellipse: $\frac{x^2}{A^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \frac{y^2}{A^2} = 1$

Formula (for foci): $c = \sqrt{A^2 - b^2}$

PROBLEM 13. Find the foci and vertices of $\frac{x^2}{4} + \frac{y^2}{9} = 1$, then graph it.

PROBLEM 14. From the WORK BOOK, section 27, do exercise 9.

***PROBLEM* 15.** From the WORK BOOK, section 27, do exercise 10.

(b) Applications

A ray shot from one focus, reflects to the other focus (and planetary orbits).

PROBLEM 16. From Textbook, section 7.5: Problems 316

***PROBLEM* 17.** From Textbook, section 7.5: Problems 317

4. Hyperbolas

(a) Typical forms of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{has asymptotes } y = \pm(b/a)x$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{has asymptotes } y = \pm(a/b)x$$

Formula (for foci): $c = \sqrt{a^2 + b^2}$

PROBLEM 18. Find the foci, vertices, and asymptotes of $\frac{y^2}{4} - \frac{x^2}{9} = 1$, then graph it.

PROBLEM 19. From the WORK BOOK, section 27, do exercise 15.

***PROBLEM* 20.** From the WORK BOOK, section 27, do exercise 16.

(b) Applications

Ray shot at one focus, reflects to the other other focus.

PROBLEM 21. Suppose two people are 100 meters apart. Describe an equation of the hyperbola shield used in order for shots at one person to be reflected to the other person.

5. Practice Problems

***PROBLEM* 22.** WORK BOOK, section 27; problems 6, 11, 12, 18, 19, 20, 21*