MTH 32 LECTURE NOTES (Ojakian)

Topic 11: Improper Integrals

OUTLINE
(References: 3.7)

1. Integrals with infinite interval
2. Intergrals and asymptotes
3. Comparison test

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1. Recall: Integrals

Theorem 1. *If $f(x)$ is continuous on $[a,b]$ then $\int_a^b f(x)\,dx$ is integrable.*

(a) Examples not covered by last case:
   i. Infinite Intervals
   ii. Discontinuities in the interval

2. Integrals: Infinite intervals

PROBLEM 1. *Consider* $\displaystyle\int_1^\infty \frac{1}{2^x}\,dx$.

(a) Why "should" this integral be finite? (pick a simple Riemann sum above it)
(b) In fact, what should be a nice upper bound?
(c) Calculate it and verify your upper bound without a calculator.

PROBLEM 2. *Evalute* $\displaystyle\int_1^\infty \frac{1}{x}\,dx$.

PROBLEM 3. *Find the volume that results from rotating the region under* $\frac{1}{x}$, *and* $x \geq 1$, *about the x-axis (Called Gabriel's Horn).*

***PROBLEM* 4.**

- *Evalute* $\displaystyle\int_2^\infty \frac{1}{x^4}\,dx$.
- *Evalute* $\displaystyle\int_{58}^\infty \frac{-7}{\sqrt{x}}\,dx$.

3. Integrals: Infinite intervals on both sides

PROBLEM 5. *Evaluate* $\displaystyle\int_{-\infty}^\infty x^2\,dx$.

***PROBLEM* 6.** *Evaluate* $\displaystyle\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

4. Integrals: Asymptote at limit of integration

PROBLEM 7. *Evaluate* $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

***PROBLEM* 8.** *Evaluate* $\displaystyle\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$

PROBLEM 9.

- *Evaluate* $\displaystyle\int_0^3 \frac{1}{x-1}\,dx$
- *And consider an incorrect way to do it!*

***PROBLEM* 10.** *Evaluate* $\displaystyle\int_{-2}^3 \frac{1}{x^4}\,dx$

5. Comparison Test

Theorem 2. *(Roughly) Suppose* $f(x) \geq g(x) \geq 0$ *on the interval* (a, b)*. Then*

(a) If $\displaystyle\int_a^b f(x)\,dx$ *converges, then* $\displaystyle\int_a^b g(x)\,dx$ *converges.*

(b) If $\displaystyle\int_a^b g(x)\,dx$ *diverges, then* $\displaystyle\int_a^b f(x)\,dx$ *diverges.*

PROBLEM 11. *For each integral determine if it converges or diverges.*

(a) $\displaystyle\int_1^\infty \frac{1}{10x+x^2}\,dx$

(b) $\displaystyle\int_{-2}^3 \frac{1+e^{-x}}{7x^4}\,dx$

***PROBLEM* 12.** *For each integral determine if it converges or diverges.*

(a) $\displaystyle\int_9^\infty \frac{1+\sqrt{x}}{x}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{x^2}{21+x^2+x^6}\,dx$

6. Practice Problems

***PROBLEM* 13.** *From* Work Book*, section 19: Problems 2a, 2b, 2c, 2d, 4a, 4b, 6a, 6b (for problem 2 and 4 also evaluate the integral).*