Topic 10: Partial Fractions

OUTLINE
(References: 3.4)

1. Preliminary Tools

    (a) Factoring Polynomials
    (b) Division of polynomials

2. Partial Fraction Decomposition

3. Integration using Partial Fraction Decompositions

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1. <u>Preliminaries</u>

    (a) <u>Factoring</u>

    PROBLEM 1. *Factor the polynomial* $(x^3 + x)(x^2 + 6x - 7)(x^2 - 1)$ *into a product of <u>linear</u> and <u>quadratic</u> polynomials.*

    PROBLEM 2. *How do you know when a quadratic factor cant be broken down further?* (Answer: If it has real zeros. Check for zeroes by either: Graphing or Quadratic Equation).

    PROBLEM 3. *Which of the following can be factored further and which cannot?*

    i. $2x^2 - 8$
    ii. $2x^2 + 8$
    iii. $x^2 - 2x + 5$
    iv. $x^2 - 10x + 25$

    PROBLEM 4. *Factor the polynomial* $(x^3 + x^2 + x)(x^2 - 2x - 15)(2x^3 - 18x)$ *into a product of linear and quadratic polynomials.*

    Fact: We can always break such polynomials down into a product of linear and quadratic polynomials.

    (b) <u>Division</u>

    i. Division fact for numbers: $\dfrac{B}{A} = ?$

    ii. Division fact for polynomials: $\dfrac{B(x)}{A(x)} = ?$

    iii. Long division for numbers: $\dfrac{467}{20} = ?$

    iv. Long division for polynomials

    PROBLEM 5. *Divide the polynomials:* $\dfrac{2x^3 - 9x^2 + 15}{2x - 5}$

    PROBLEM 6. *Divide the polynomials:* $\dfrac{x^3 + 2x}{x - 1}$

2. Preliminary: Partial Fraction Decompositions

   NOTE: Restrict to denominators with factorization into LINEAR factors.

   Goal: Write $\dfrac{\text{polynomial}_1}{\text{polynomial}_2} = \dfrac{\text{number}_1}{\text{simple polynomial}_1} + \dfrac{\text{number}_2}{\text{simple polynomial}_2} + \cdots$

   PROBLEM 7. *Find the Partial Fracion Decomposition of* $\dfrac{4x}{x^2 - 2x - 8}$

   Method:

   (a) If necessary divide (if top degree $\geq$ bottom degree)

   (b) Factor the denominator (i.e. the bottom).

   (c) For <u>each factor</u> $\mathcal{F}$ on the bottom create a fraction-to-be:
   - Make the bottom $\mathcal{F}$ (for repeated factors, increment the exponents).
   - Make each top a new variable
   - Solve for the letters A, B, C, etc. by either 1) Clearing the fractions, or 2) Ingenius substitution.

   PROBLEM 8. *Find "the form" for various given Partial Fraction Decompositions.*

   PROBLEM 9. *Find the Partial Fracion Decomposition of* $\dfrac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x}$

   PROBLEM 10. *Find the Partial Fracion Decomposition of* $\dfrac{x^5 + 2x^4 + 1}{x^3 + x^2}$

3. Integration by Partial Fraction Method

   PROBLEM 11. *Do integrations corresponding to above ...*

   PROBLEM 12. *Evaluate* $\displaystyle\int \dfrac{x^3 + x}{x - 1}\, dx$

4. Practice Problems

   PROBLEM 13. *From* Work Book, *section 17: 1a, 1b, 1c*