

# MTH 32 LECTURE NOTES (Ojakian)

## Topic 9: Trig Substitution

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### OUTLINE

(References: 3.3)

1. Preliminary Tools
  2. Trigonometric Substitution
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#### 1. Preliminaries

##### (a) Trig Identities

- i.  $\sin^2 \theta + \cos^2 \theta = 1$
- ii.  $\sin(2\theta) = 2(\sin \theta)(\cos \theta)$
- iii.  $\cos(2\theta) = 1 - 2\sin^2 \theta$

**PROBLEM 1.** *Textbook section 3.3: exercise 126.*

##### (b) Inverse Trig Functions

**PROBLEM 2.** *Calculate the following:  $\sin(\pi/2)$ ,  $\arcsin(1)$ ,  $\arcsin(-1)$*

**PROBLEM 3.** *Suppose  $\sin \theta = x/7$ . Express  $\theta$  in terms of  $x$ .*

##### (c) Trig and the Pythagorean Theorem

**PROBLEM 4.** *Suppose we have a triangle with an angle  $\theta$  such that  $\sin \theta = x/7$ .*

*Find  $\cos \theta$ ,  $\sec \theta$ , and  $\csc \theta$ .*

*Express the following in terms of  $x$ :  $10(\sin \theta)(\cos \theta)$*

##### (d) Some integration formulas for other trig functions ...

- i. Recall  $\int \sin$  and  $\int \cos$
- ii.  $\int \csc x = -\ln |\csc x + \cot x|$
- iii.  $\int \sec x = \ln |\sec x + \tan x|$

**PROBLEM 5.** *Evaluate  $\int \csc 3x \, dx$*

##### (e) Exercises

**\*PROBLEM\* 6.** *Simplify  $\sqrt{25 - 25\sin^2 \theta}$  to a single trig function.*

**\*PROBLEM\* 7.** *Calculate the following:  $\sin(\pi/4)$ ,  $\arcsin(0)$*

**\*PROBLEM\* 8.** *Suppose  $\sin \theta = x$ . Express  $\theta$  in terms of  $x$ .*

**\*PROBLEM\* 9.** *Suppose we have a triangle with an angle  $\theta$  such that  $\sin \theta = x/5$ .*

*Find  $\cos \theta$ ,  $\tan \theta$ , and  $\cot \theta$*

**\*PROBLEM\* 10.** *Evaluate  $\int \sec(-5x) \, dx$*

2. Trigonometric Substitution

Integrals with expression like the following:  $\sqrt{a^2 - x^2}$  (use  $x = a \sin \theta$ )

**PROBLEM 11.** Evaluate  $\int \sqrt{9 - x^2} dx$

**PROBLEM 12.** Find the length of the curve  $y = \sqrt{36 - x^2}$  between  $x = -1$  and  $x = 1$

**PROBLEM 13.** From Textbook Section 3.3: 145, 153

3. Practice Problems

**\*PROBLEM\* 14.** From Textbook, section 3.3 (page 296ff): 134, 136, 138