MTH 32 LECTURE NOTES (Ojakian)

Topic 6: Exponential Growth and Decay

OUTLINE
(References: 2.8)

1. Exponential Growth: Concept.
2. Exponential Growth/Decay Applications: population growth, epidemic spread, interest, radioactive decay.

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1. Exponential Growth: Background

PROBLEM 1. *Consider a human population. Initially one person is infected with a disease. Every day, each person passes on the disease to one new person.*

Let $P(t)$ = the number of infected people after t days. Answer the following questions:

(a) Make a table with t versus $P(t)$.
(b) Graph $P(t)$ as best you can.
(c) What is the ratio of $P'(t)$ to $P(t)$?
(d) Consider what the last point means for disease spread.

PROBLEM 2. *With reference to the above example answer the following questions.*

(a) Why does it make sense that $P'(t)$ increases as $P(t)$ increases?
(b) Why does it make sense that $P'(t)/P(t)$ is constant?
(c) What are examples of phenomenon that **would** *have the above properties? What are examples of phenomenon that* **would not** *have the above properties?*

Q: What does it mean to "*grow at a rate proportional to your size*"?

PROBLEM 3. *Consider a function $y(t)$ such that $y'(t) = k \cdot y(t)$ (where k is some constant called the "growth constant"). Answer the following questions:*

(a) What is the initial value of $y(t)$?
(b) What is the growth rate of $y(t)$?

PROBLEM 4. *"Solve": $y'(t) = k \cdot y(t)$.*

Note some functions that work and ones that do not.

PROBLEM 5. *Suppose a disease is spreading at an exponential rate (with growth rate 3 when time is measured in weeks), where initially 10 people are infected.*

(a) How many people are infected after 28 days?
(b) When will a million people be infected?

PROBLEM 6. *From Textbook, Section 2.7: Exercise 356*

PROBLEM 7. *Note that the doubling rate is constant. Consider the last problem and use that point to quickly determine when the population will reach 20 million.*

2. Continuously Compounding Interest

PROBLEM 8. *First consider starting with 1000 dollars and do the following compounding: Once a year, twice a year, quaterly.*

And then extend to multiple years.

PROBLEM 9. *Repeat the last problem but now with "continuous compounding" and bring in an alternate definition of e.*

PROBLEM 10. *From Textbook, Section 2.7: Exercise 365. But first do it if you get an annual interest rate that is compounded twice a year.*

3. Radioactive decay and Carbon Dating

PROBLEM 11. *Consider the exponential process with a negative "growth constant" ... now called "decay constant". Now what is our solution?*

Note how it goes to zero faster than many functions.

PROBLEM 12. *What natural phenomena follow "exponential decay"?*

PROBLEM 13. *Contrast the "doubling" property of exponential growth with the "half-life" property of exponential decay.*

PROBLEM 14. *From Textbook, Section 2.7: Exercises 354, 369*