

## Topic #20 (Math 31)

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- Goals (Section 4.10)
  - Anti-derivatives
  - Initial Value Problems
- Anti-derivative Definition
  - The anti-derivative of  $f(x)$  is a function  $F(x)$  such that  $f(x) = F'(x)$
  - Way to think of anti-derivative: The reverse of differentiation
  - Examples of Verifying – SECTION 4.10: 465 – 469
- Examples (do not use formulas!):
  - $e^x$ ,  $\sin(Kx)$ ,  $\cos(Kx)$
  - constants; zero
  - powers of  $x$ ; include fractional powers
  - addition of functions
- Antiderivative Theorem
  - Observe: Multiple anti-derivatives.
  - Theorem: Suppose  $F(x)$  is an antiderivative of  $f(x)$ . Then
    - $F(x) + C$  is also an antiderivative, for every constant  $C$ .
    - Furthermore,  $f(x)$  has no other antiderivatives.
- Notation and terminology
  - If  $F(x)$  is an antiderivative of  $f(x)$ , then  $F(x) + C$  is called the “most general antiderivative of  $f(x)$ ” or the “indefinite integral of  $f(x)$ ”
  - Write  $\int f(x)dx = F(x) + C$  “ $F(x) + C$  is the most general antiderivative of  $f(x)$ ”
  - Distinguish “an antiderivative” from “the most general antiderivative”.
  - Verification example:
    - $\int \sqrt{2x+1} dx = \left(\frac{1}{3}\right)(2x+1)^{\frac{3}{2}} + C$
  - Write some of previous examples in notation.
  - Picture of multiple anti-derivatives (page 488, Figure 4.85)
- Formulas
  - See table on Page 489 (Figure 4.13).
  - See Page 492 – Theorem 4.16
- Examples
  - Antiderivative of  $\sin(5x)$ ,  $\cos(5x)$
  - Antiderivative of  $e^{5x}$
  - SECTION 4.10: 470 – 498
- Initial Value Problems
  - Example: Find the “function” whose slope is always 2?? ... (okay: and hits the  $y$  axis at 1)
  - Examples: do some of above with an initial value.
  - Note: It is finding the “function” that satisfies the equation.
  - Examples – SECTION 4.10: 499 – 503
- Application to displacement, velocity, and acceleration.
  - Have opposite relationship from below...
  - Examples – SECTION 4.10: 509 – 514