Topic #19 (Math 31)

1. Goals (Section 4.9): Newton's Method
    a. Use: To approximate roots.
2. Idea: See picture on page 473
3. Newton's Method Steps
    a. Pick starting approximation: X0
    b. Repeatedly apply Newton's formula: $X_n = X_{n-1} - f(X_{n-1}) / f'(X_{n-1})$
    c. Stop when successive approximations "look close"
4. Examples:
    a. Example 4.46 (page 474)
    b. Example 4.47 – Finding Square Root – page 475
5. Why it works
    a. Find tangent line at X0 (can use linear approximation formula)
    b. X1 is a root, so it makes the tangent line value = 0
    c. Solve for X1
    d. Then consider repeating the process.
6. Failures of Newton's Method
    a. Derivative = 0
    b. Approach "wrong root" (picture above example 4.48; top of pag 477)
    c. Non-convergence to any root (Example 4.48, page 477)
7. Examples
    a. Determine "graphically" for some pictures.
    b. Just Newton Formula: 406 – 410
    c. Full Newton: 422 – 431
    d. Newton to find max/min: 438 – 445
    e. Apply Newton's Method to lines (including horizontal)
    f. Examples of failure:
        i. Apply to square root
        ii. Apply to third root
        iii. Explain failure with start of 1 on x^3 – 3x + 6 = 0