Topic #16 (Math 31)

1. Goals (Sections 4.5, 4.6)
    a. Concavity and second derivative test
    b. Putting it all together into a detailed graph!
2. Recall all the info we can get about graph already
    a. X and Y intercepts (algebra)
    b. Recalling domain (algebra)
    c. Limits at number and at infinity
    d. Increasing/decreasing info (first derivative)
    e. Absolute extrema (first derivative)
    f. Local extrema (first derivative test)
    g. NEXT: How does the graph bend?
        i. Concave up (part of valley)? (think CU – is upright for a cup)
        ii. Concave down (part of hill)?
        iii. Inflection Point: Change in concavity.
            1. Example: Price, inflation, and inflection point.
            2. Example: Economic growth, and inflection point.
            3. Inflection point (colloquial): "A moment of dramatic change, especially in the development of a company, industry, or market."
3. Bring in second derivatives:
    a. Take the derivative twice.
    b. Examples: Any polynomial.
4. Geometric Interpretation
    a. Keep in mind:
        i. Function g(x) increase correspond to g'(x) positive
        ii. Function g(x) decrease correspond to g'(x) negative
    b. Keep in mind g'(x) is itself a function.
    c. Thus: g''(x) is the derivative of g'(x) so
        i. g'(x) increase correspond to g''(x) positive
        ii. g'(x) decrease correspond to g''(x) negative
    d. Example: Recall this in picture.
    e. Example: In basic parabola y = x^2 – 3
    f. In general: What shape corresponds to f''(x) positive and which for f''(x) negative?
        i. Concave up (CU): Derivative increasing (second derivative positive)
        ii. Concave down (CD): Derivative decreasing (second derivative negative)
5. Concavity Test (analogous to first derivative corresponding to increasing/decreasing)
    a. F''(x) positive corresponds to concave up
    b. F''(x) negative corresponds to concave down
    c. Examples. Do some of Section 4.5: 224 – 230 (just concavity – below with inflection points)
6. Inflection Point
    a. An x value where the function is continuous, AND
    b. Concavity changes

c. Challenge: draw graph with concavity change at “a” but no Inflection Point at “a”
d. Examples.
    i. Section 4.5: Exercise 196
    ii. Section 4.5: Exercises 211 – 215 (from pictures)

7. Technique for determining concavity and inflection points
    a. Find F’’(x)
    b. Find numbers where F’’(x) = 0 or F’’(x) DNE
    c. Consider the intervals produced by the these numbers
    d. Determine of the sign of the second derivative on each interval
    e. Apply the concavity test.
    f. Check for Inflection Points
    g. Examples. Do some of Section 4.5: 224 – 230 (bring in inflection points)
8. Issues to recall
    a. End behavior
        i. 4 possibilities: +/- infinity, DNE, a number
        ii. Notation: lim
        iii. Examples via picture.
        iv. Asymptote.
        v. Example of Polynomials: Always + infinity or – infinity (highest power determines behavior)
        vi. Example of 2 + 1/x, which is 2.
        vii. Example of sin: DNE
    b. Vertical Asymptotes
        i. Look for when we divide by zero.
9. Examples of graphing
    a. Section 4.6: Exercises from 294 – 305
10. Inflection points and velocity.
    a. Draw graph.
    b. If f(t) is distance, f’(t) is velocity, and f’’(t) is acceleration.
    c. Linear situation: Velocity constant <-> f’(t) constant <-> f’’(t) zero <-> acceleration 0
    d. CU situation: Velocity increasing <-> f’(t) increasing <-> f’’(t) positive <-> acceleration +
    e. CD situation: Velocity decreasing <-> f’(t) decreasing <-> f’’(t) neg <-> acceleration neg
    f. IP: Change from increasing velocity to decreasing velocity or vice versa.
11. In general:For any function f(x)
    a. Recall: f’(x) is the rate of change of f.
    b. COPY UNDER ABOVE
        i. Linear situation: f’s rate of change is constant …
        ii. CU situation: f’s rate of change increasing <-> f’(t) increasing <-> f’’(t) positive
        iii. CD situation: f’s rate of change decreasing <-> f’(t) decreasing <-> f’’(t) neg
        iv. IP: Change from increasing rate of change to decreasing rate or vice versa
    c. Examples. Section 4.5 exercises among 241 – 245 (interpretation)