Topic #15 (Math 31)

1. Goals (Section 4.3, part of 4.5)
    a. Maxima and Minima
    b. First Derivative Test
    c. Rough Sketches of Graphs
2. Overview of important direction
    a. Determining the "shape" of a graph
    b. Aspects of shape
        i. Where is graph increasing? Where decreasing?
        ii. Where are the maximum and minimum points?
    c. Key application: Optimization problems- Finding the maximum or minimum
3. From Graph: Fully describe where the graph is increasing and where decreasing, max, min.
    a. Use Examples: $y = |x| + 2$ and $y = -x^2 + 1$
    b. Increasing/Decreasing definition on interval (use strict increase/decrease)
    c. Absolute Extrema (i.e. Max, Min). Larger than or equal to all other values for max.
    d. Local Extrema (i.e. Max, Min).
        i. Local maximum refers to the top of a hill; i.e. point where all nearby points are at or below.
        ii. Local maximum value is the y value of such a hill
        iii. The local maximum occurs at the corresponding x value.
        iv. Local minimum refer to the bottom of a valley; i.e. all nearby points are at or above.
    e. Function terminology:
        i. "Value" of a function refers to "y value", i.e. the value along the vertical axis.
        ii. "at" "when" "where": refers to the "x value", i.e. the value along the horizontal axis.
    f. Examples:
        i. Section 4.5: 201 – 210 (from pictures)
        ii. Section 4.3: 104, 107 (draw picture with property)
4. Technique for Absolute Extrema:
    a. Problem: Find the absolute extrema of $f(x)$ on the interval $[a, b]$
    b. Basic Fact: $f(x)$ has Local Extrema at x (inside interval) IMPLIES derivative = 0 or DNE
    c. Find the derivative $f'(x)$
    d. Find the "critical numbers": Zero derivative or derivative not exist
    e. Evaluate f at the critical numbers and at "a" and at "b". The smallest is the min, largest the max.
    f. Watch out: If one of "a" or "b" is infinity!

5. Examples:
    a. Section 4.3: Some from 108 – 117 (finding critical points)
    b. Section 4.3: Some from 118 – 134 (just finding absolute extrema)
    c. Section 4.3: 140 (application)
    d. Section 4.3: Some from 144, 145 (piecewise defined)
    e. Section 4.3: 90 (a proof)
6. Technique for finding local extrema – ***<u>First Derivative Test</u>***
    a. Consider the intervals produced by the critical numbers
    b. Determine the sign of the derivative on each interval (method that works for us: Use test points)
    c. Use sign change to determine if critical point is local max, min, or neither.
        i. If derivative changes from (+) to (–) at c, then there is a local max at c.
        ii. If derivative changes from (–) to (+) at c, then there is a local min at c.
    d. Examples:
        i. Section 4.5: Exercises from 224 – 230, ignoring (c) and (d), but giving “rough sketches” where feasible.