Kerry Ojakian's MTH 31 Class
Handout #1

LIMIT FACTS.

1. If $f(x)$ is continuous at a (e.g. polynomial, rational functions where defined) and a is a real number, then $\lim_{x\to a} f(x) = f(a)$ (i.e. *direct substitution*).
2. $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$
3. For any constant c, $\lim_{x\to a} c = c$

FUNDAMENTAL FUNCTIONS.
In all the expressions below, k is a positive integer (e.g. 1, 2, 3, etc) and a is any real number or $+\infty$ or $-\infty$.

4. $\lim_{x\to +\infty} x^k = +\infty$
5. $\lim_{x\to -\infty} x^k = \begin{cases} +\infty & \text{if } k \text{ is even} \\ -\infty & \text{if } k \text{ is odd} \end{cases}$
6. $\lim_{x\to +\infty} \frac{1}{(x-a)^k} = 0$
7. $\lim_{x\to -\infty} \frac{1}{(x-a)^k} = 0$
8. $\lim_{x\to a^+} \frac{1}{(x-a)^k} = +\infty$
9. $\lim_{x\to a^-} \frac{1}{(x-a)^k} = \begin{cases} +\infty & \text{if } k \text{ is even} \\ -\infty & \text{if } k \text{ is odd} \end{cases}$
10. $\lim_{x\to a} \frac{1}{(x-a)^k} = \begin{cases} +\infty & \text{if } k \text{ is even} \\ DNE & \text{if } k \text{ is odd} \end{cases}$

LIMIT LAWS.
Below, let $f(x)$ and $g(x)$ be functions. Laws true *if all limits exist.*

11. $\lim_{x\to a}(f+g)(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
12. $\lim_{x\to a}(f-g)(x) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
13. $\lim_{x\to a}(f\cdot g)(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
14. $\lim_{x\to a} \left(\frac{f}{g}\right)(x) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, if $\lim_{x\to a} g(x) \neq 0$.
15. If f and g are continuous everywhere then $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$